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A long-run Pure Variance Common Features model for the common volatilities of the Dow Jones

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Abstract

In this paper, a new model to analyze the comovements in the volatilities of a portfolio is proposed. The Pure Variance Common Features model is a factor model for the conditional variances of a portfolio of assets, designed to isolate a small number of variance features that drive all assets' volatilities. It decomposes the conditional variance into a short-run idiosyncratic component (a low-order ARCH process) and a long-run component (the variance factors). An empirical example provides evidence that models with very few variance features perform well in capturing the long-run common volatilities of the equity components of the Dow Jones.

JEL classification: C52; C32

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1. Introduction

In finance, there is a strong belief that movements in the price of one particular asset are quite likely to coincide with movements in the prices of other assets, possibly quoted in different markets. These comovements might be caused by the reaction of economic agents to particular changes in some macroeconomic and financial variables or, maybe, to specific news about the company or about the economic sectors involved. In addition, the movements in one asset price may have implications that are likely to affect the value of other assets either contemporaneously or with some lags. This behavior has traditionally been modeled with factor models in which asset prices are driven by a small number of latent variables called factors and others named idiosyncratic disturbances. The concept of factors plays a crucial role in two major asset pricing theories: the mutual fund separation theory of which the standard Capital Asset Pricing Model (CAPM) is a special case and the Arbitrage Pricing Theory (APT) of Ross (1976, 1978). Typically, these models are linear and are identified by the assumption that all these latent variables are independent. Their aim is to seek a data reduction by specifying a small number of latent variables that influence a large number of output variables.

Common Features (CF), introduced by Engle and Kozicki (1993), is a further generalization of these concepts. A small number of latent variables, with a specific characteristic or feature, influence all the observables and give them this feature, with respect to which, the problem becomes of smaller dimension and more tractable. By associating the factors with such features, it is possible to build factor style models for much more general situations. In many cases, these factor models can be formulated as reduced rank regressions or canonical correlation problems. The most widely used example of CF and the main motivation for the idea is cointegration, the phenomenon where a reduced number of common stochastic trends can determine the long-run properties of a large number of observables (Granger, 1983; Engle and Granger, 1987). There are many approaches to estimation and test for the number of unit roots, but the most popular are based on reduced rank regression and on canonical correlations, as in Johansen (1988) and Ahn and Reinsel (1990). Many other types of CFs have been examined in the literature, such as serial correlation CFs (Vahid and Engle, 1993, 1997) which are called common cycles in macroeconomics and risk premiums in finance, common seasonals (Engle and Hylleberg, 1996; Cubadda, 1999), common non-linearities (Anderson and Vahid, 1998), or common structural breaks (Hendry and Mizon, 1998). In particular, there are a few CFs that examine the structure of the second moments of a set of variables such as common ARCH factors (Engle and Susmel, 1993), common persistence (Bollerslev and Engle, 1993), or common long-range dependence (Ray and Tsay, 2000). All these structures have the potential and in some cases the realized benefit of improving the performance of large models by restricting the number of parameters to ensure that such features are common.

Traditionally, since the seminal papers by Engle (1982) and Bollerslev (1986), volatility is modeled with univariate ARCH/GARCH models. Nevertheless, since the beginning of this burgeoning literature both financial econometricians and practitioners have understood the importance of multivariate GARCH models because the finance practice needs to handle the risk involved in big (if not huge) portfolios. Among these Multivariate GARCH models, the most important are the VECH model of Bollerslev et al. (1988), the Constant Conditional Correlation (CCC) model of Bollerslev (1990), the Factor ARCH model of Engle et al. (1990), and Ng et al. (1992), the BEKK model by Engle and Kroner (1995), or the more recent Dynamic Conditional Correlation (DCC) model of Engle (2002).

The typical difficulty with these models is the number of parameters required to specify large covariance matrices. Many of the important simplifications are factor models—such as the Factor ARCH models of Engle et al. (1990) or the conditionally heteroskedastic latent factor models of Diebold and Nerlove (1989). Intuitively, the Factor ARCH model assumes that there are few factors or portfolios (i.e. linear combinations of the observed random variables) whose time-varying variances drive the whole covariance matrix of the system. On the other hand, the conditionally heteroskedastic latent factor model of Diebold and Nerlove (1989) is a traditional statistical factor analysis model, with a diagonal idiosyncratic covariance matrix, in which the variances of the common factors are parameterized as univariate ARCH processes. Sentana (1998) highlights the basic differences between these two models. The covariance matrix of the Factor ARCH model is by construction measurable with respect to the usual information set that contains only past values of the observables, while the conditionally heteroskedastic latent factor model can be regarded as a stochastic volatility model. Furthermore, another distinctive feature is related to the implicit definition of the factors which is completely different between the two models. In conditionally heteroskedastic latent factor models, the factors capture the comovements between the observed series, whereas in Engle et al. (1990) Factor ARCH model, the factors are directly related to those linear combinations of the observed series which summarize the comovements in their conditional variances. King et al. (1994) generalize the Diebold and Nerlove's (1989) model by constructing a multivariate factor model that nests the latter, in which time-varying volatility of returns is induced by the changing volatility of the underlying factors, that can be observable or unobservable. A great advantage of their model is not only a parsimonious representation of the conditional variance–covariance matrix of excess returns as a function of the changing variances of a small set of factors, but also the easier identification of these factors in this context. Actually, Sentana and Fiorentini (2001) show how identification problems in factor models with conditional heteroskedasticity can be easily solved when variation in factor variances is accounted for in the estimation.

In this paper, we suggest a new type of CFs. A Pure Variance Common Features (PVCF) is a statistical model that describes how the conditional variances of a collection of assets may all depend upon a small number of variance factors. This differs from the factor models described above in that it does not require that the covariances also depend on these same factors. This is precisely the problem that the risk manager of an options portfolio faces, and is also a central feature of measuring risk in standard portfolio problems. The second extension of the volatility factor models is that the idiosyncrasies are allowed to have short-run variability. The factors explain common movements in long-run volatilities. The existence of such common components implies that the relationships between the volatilities are tied together in the long run, and therefore are interpretable as long-run equilibrium relationships. As with other types of CFs, we should be able to obtain superior volatility forecasts by using the fact that there exist few common volatility components (or PVCF). Furthermore, the presence of few common volatility components can have important implications for asset pricing relationships and in optimal portfolio allocations. The price of an asset typically depends on the conditional covariance with some benchmark portfolio. Therefore, the pricing of long-term contracts may be completely different from that of one-period contracts if there are common long-run volatility components in the conditional variance or in the covariance with the benchmark portfolio. Last but not least, the pricing of certain portfolios of assets can be more sensitive to these long-run volatility components than to the idiosyncratic short-run volatility components.

The plan of the rest of the paper is as follows. Section 2 illustrates the options portfolio problem that motivates the model. Section 3 introduces the PVCF model that should be useful in building big models and managing portfolios of options. Section 4 develops the econometric specification and the problems involved in the detection of common long-run volatility components. The empirical relevance of the PVCF model is discussed in Section 5 where an application to the thirty stocks of the Dow Jones Industrial Average Index is presented. The PVCF model seems to perform rather well by identifying two or three PVCF that affect all the volatilities. Section 6 concludes giving also directions for further research.

2. Measuring the risk of an options portfolio

Risk managers, options traders and strategists must understand the risk of an options portfolio. In general, this would include options with several strikes, different maturities and various underlying assets. By invoking some variant of Black Scholes option pricing, it is easy to evaluate the risk of portfolios of options with a single underlying asset. In this fashion, options traders aim to reduce risk by holding portfolios that are both delta and vega neutral, so that they are approximately unaffected by small movements in the underlying asset price and in its volatility. With multiple underlyings, only the deltas are typically evaluated.

Consider a collection of options whose prices at time t are given by a vector p_t. The price of the underlying assets, arranged in the same fashion, is given by s_t and the volatilities of these assets can be stacked into a vector v_t. Some of these volatilities will be the forecast volatility over a short horizon while others will be over a long horizon. For many volatility models, these will simply be proportional.

The delta of this portfolio of options is defined by

$$\Delta_t = \frac{\partial p_t}{\partial s_t'}. \tag{1}$$

Most elements of this matrix are zero since the price of an option on one asset will be unaffected by a change in the value of another underlying asset price as long as the first is unchanged. There may be additional parameters in delta, and each of these must be evaluated at the time the hedge or risk measure is undertaken; for example, the estimate at $t-1$ would be written $\Delta_{t/t-1}$. A portfolio with w dollars in each position would be valued at $\tilde{\pi}_t = w'_{t-1}p_t$. To make this portfolio delta neutral, offsetting positions would be taken in the underlying assets to give portfolio value

$$\pi_t = w'_{t-1}(p_t - \Delta_{t/t-1}s_t). \tag{2}$$

This portfolio has no risk for small movements in any of the components of p_t.

When volatility changes the option prices will change. The vega of the vector of options is defined as

$$\Lambda_t = \frac{\partial p_t}{\partial v'_t}. \tag{3}$$

Again, this would be expected to be a block diagonal matrix. Since the derivative of s_t with respect to v_t is zero, the vega of the delta neutral portfolio in (2) is

$$\frac{\partial \pi_t}{\partial v'_t} = w'_{t-1}\Lambda_{t/t-1}, \tag{4}$$

where $\Lambda_{t/t-1}$ denotes the estimate of Λ_t at $t-1$. By the chain rule, the derivative with respect to the conditional variance is

$$\frac{\partial \pi_t}{\partial v_t^2} = \frac{1}{2} w'_{t-1}\Lambda_{t/t-1}D_t^{-1}, \tag{5}$$

where v_t^2 is the vector of conditional variances and D_t is the diagonal matrix of conditional standard deviations. If we denote $\tilde{w}_{t-1} = w_{t-1}/2$ and the variance–covariance matrix of the volatilities can be forecast as

$$Var_{t-1}(v_t^2) \equiv \Psi_t, \tag{6}$$

then the portfolio variance is given by

$$Var_{t-1}(\pi_t) = \tilde{w}'_{t-1}\Lambda_{t/t-1}D_t^{-1}\Psi_t D_t^{-1}\Lambda'_{t/t-1}\tilde{w}_{t-1}. \tag{7}$$

Only if $\tilde{w}'\Lambda = 0$, will this portfolio not be dependent on the covariance matrix of volatilities. This can be achieved by balancing the volatility exposure with respect to each of the underlyings. Often, this is not possible, leading to a need for a covariance matrix of the volatilities of the underlyings. This expression gives quantitative meaning to the sense in which a short volatility position in one asset can be hedged by a long volatility position in another. If the volatilities are highly correlated then the risk will be small.

The focus of the paper is on developing expressions for the covariance of asset volatilities as indicated in (6). From the expression, it appears that this is the forecast of the volatility over the next day, however from the development, it should be clear that this is a forecast of the volatility over the remaining life of an option, so it will generally be many days or even years. For long horizon forecasts, the volatility becomes very small as the new information has a relatively small effect on the long-run forecasts.

In the next section, a factor model will be introduced for the conditional variances. This will provide a method for calculating the conditional covariance matrix among a set of volatilities.

3. The PVCF model

An important problem in a wide range of financial applications is the modeling of the variance–covariance matrix of a high number of assets. This requires estimation not only of the variances, but all the covariances. The Factor ARCH model introduced by Engle et al. (1990) parameterizes this matrix in terms of a small set of factors with time-varying variances. Although there are data sets where one or two factors describe the entire covariance matrix, this might not always be the case.

Instead, we can look for CFs that only affect the variances. The first step in many approaches for the estimation of a covariance matrix is to estimate the univariate variances, as in Engle's (2002) DCC model. While it is possible to estimate many variances separately, as if they were independent series, there may be relations between these variances that can and should be exploited. Frequently, simple GARCH models of a collection of assets show remarkable similarities possibly due to the presence of common volatility processes. While a full model of portfolio allocation and Value at Risk will require estimating the correlations, a closely related problem will depend dramatically on the relations between the variances.

Consider a vector of asset excess returns, $\tilde{r}_t \in \mathbb{R}^N$, with conditional mean vector μ_t. To simplify the notation consider the $N \times 1$ vector $r_t = \tilde{r}_t - \mu_t$ corrected to have zero mean by subtracting the conditional mean vector.[1] Then, construct a vector of the squares of these returns denoted $r_t^2 = r_t \odot r_t$ where $\odot$ represents the Hadamard (or element by element) product. Such a vector would be equivalent to the vector of the $diag\{r_t r_t'\}$, where $diag\{A\}$ represents a column vector extracted from the main diagonal of matrix A. Based on a sigma field of past values of all returns ($\Im_{t-1}$), the problem is to specify and estimate the full variance–covariance matrix

$$V_{t-1}(r_t) \equiv H_t = D_t R_t D_t \tag{8}$$

or the single conditional variances

$$E_{t-1}(r_t^2) \equiv h_t = diag\{H_t\} = diag\{D_t^2\}, \tag{9}$$

where H_t is the covariance matrix of r_t, D_t is the diagonal matrix of conditional standard deviations and R_t is the correlation matrix.

[1] We can also consider a more general setting where μ_t is a vector of time-varying risk premia, related to the factors that drive the return process. As explained later in the paper, considering μ_t as a linear combination of factor risk premia, the PVCF model can be translated into an APT framework. However, the focus of the paper is on isolating common volatilities and we leave further analyses exploring this more general model for future work.

A PVCF model for this problem can be formulated as a linear factor model for the conditional variances of r_t

$$h_t = \Gamma\xi_t + diag\{\Omega_t\}, \tag{10}$$

where ξ_t is a $K \times 1$ vector of positive random variables (called variance factors), Γ is an $N \times K$ matrix of variance factor loadings, and Ω_t is an $N \times N$ diagonal positive semi-definite matrix of idiosyncratic variances that in the literature are usually assumed to be constant. The variance–covariance matrix of this vector of variances can be directly evaluated from (10) when the idiosyncratic covariance matrix is constant

$$V_{t-1}(h_{t+1}) = \Gamma V_{t-1}(\xi_{t+1})\Gamma'. \tag{11}$$

Notice that h_t is given as a function of information at time $t-1$, but the value of h_{t+1} is a random variable with a covariance matrix as summarized above.

This formulation is closely related to the CAPM and APT asset pricing models as well as to the Factor ARCH model. An APT model with K factors can be expressed as

$$r_t = \Gamma f_t + \eta_t, \quad E_{t-1}(f_t\eta_t') = 0, \tag{12}$$

where returns and factor returns are interpreted as excess returns. The covariance matrix of this vector of returns is

$$V_{t-1}(r_t) = \Gamma V_{t-1}(f_t)\Gamma' + \Omega_t, \quad V_{t-1}(\eta_t) \equiv \Omega_t. \tag{13}$$

If the idiosyncratic covariance matrix is time invariant, then all variances and covariances of returns will depend only on the covariance matrix of the factors. If, in addition, the factors are conditionally uncorrelated, then the variances of the factors will be the only state variables. Thus, the CF described in (10), are the factor variances. The covariances among volatilities will depend on the variance–covariance matrix of the conditional variances as in (11).

If idiosyncratic volatilities are not constant, then there will be time variation in h_t beyond that explained by the factors. For most asset management functions, transitory changes in volatilities can be ignored. Thus, if the idiosyncratic volatilities are mean reverting at a rapid rate, then the model can be treated as a factor model. We here introduce the idea of a long-run pure variance common feature (LRPVCF), which is closely related to the concept of copersistence suggested by Bollerslev and Engle (1993). It allows the possibility of short-run volatility in the idiosyncrasies.

We assume a low-order ARCH process for the idiosyncratic variances. These assumptions guarantee that each element of $diag\{H_t\}$ is positive and can be written as

$$\{H_t\}_{ii} = h_{it} = \gamma_i\xi_t + \alpha_{i0} + \sum_{j=1}^{J}\alpha_{ij}r_{i,t-j}^2, \tag{14}$$

where γ_i is the ith row of Γ, a simple ARCH(p) model is assumed for the idiosyncratic variances, ξ_t is a vector of K positive variance factors, and the α_{ik}'s are non-negative parameters. In addition, it is expected that $K \ll N$, or, alternatively, that the number of variance factors which drive the comovements in the conditional variances of the whole portfolio is quite small.

An alternative useful formulation of the additive model in (10) is a vector multiplicative model such as

$$h_t = \exp[\Gamma \xi_t^* + diag\{\Omega_t^*\}], \tag{15}$$

where, $\xi_t^* = \log(\xi_t)$, and Ω_t^* are the Exponential ARCH equivalents of the matrix Ω_t. With this multiplicative formulation, the logarithm of the conditional covariance matrix has now a factor structure. Each element of the main diagonal of the conditional covariance matrix can therefore be written as

$$\{H_t^*\}_{ii} = \log(h_{it}) = \gamma_i \xi_t^* + \alpha_{i0} + \sum_{j=1}^{p} \alpha_{ij} \left| \frac{r_{i,t-j}}{h_{i,t-j}^{1/2}} \right| + \sum_{j=1}^{p} \delta_{ij} \frac{r_{i,t-j}}{h_{i,t-j}^{1/2}}. \tag{16}$$

The LRPVCF model considers also time-varying idiosyncratic volatilities with low persistence and, therefore, it is not possible to construct portfolios with constant conditional variances as in the Factor ARCH model.

Usually, one of the main purposes in building a new model is to have better multiperiod forecasts. In the additive PVCF model, the multiperiod forecasts of the conditional variances in the main diagonal of H_t can be calculated as follows

$$E_t(diag\{H_{t+\tau}\}) = \Gamma E_t(\xi_{t+\tau}) + E_t(diag\{\Omega_{t+\tau}\}), \tag{17}$$

where the variance factors $\xi_{t+\tau}$ are forecastable through the model adopted to get the factors themselves, while the idiosyncratic variances come from a low-order ARCH process. The τ-period ahead forecast for the ith asset's conditional variance will be

$$h_{i,t+\tau} = \gamma_i E_t(\xi_{t+\tau}) + \alpha_{i0} + E_t\left(\sum_{j=1}^{p} \alpha_{ij} r_{i,t+\tau-j}^2\right). \tag{18}$$

For long horizon forecasts the last term is constant, leaving the volatility process as an exact factor model. A parallel forecast for the multiplicative form is similar, but requires some distributional assumptions.

4. Econometric specification and estimation

To complete the econometric specification of the LRPVCF model, we must specify the joint distribution of the factors and the returns. The easiest specification is when the factors are observables. The underlying factors may be the conditional variances of observable indices such as the Dow Jones, the S&P500 or the NASDAQ. In this case, the volatility of these indices is estimated with a univariate GARCH model and in each case an asymmetric component model is chosen. In some versions of the model, the observed implied volatility of the S&P100 as measured by the new VIX index is used instead of the GARCH volatility of the underlying. This version of the model is called Market PVCF (PVCF-MKT) model.

Assuming joint conditional normality both for the returns and the factors, we can write the full model as

$$\begin{pmatrix} r_t \\ f_t \end{pmatrix} \Bigg| \Im_{t-1} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} D_t^2 & G_t \\ G_t' & F_t \end{pmatrix} \right), \tag{19}$$

where

$$diag\{D_t^2\} \equiv h_t = \Gamma \xi_t + \omega_0 + \sum_{j=1}^{p} \omega_j \odot r_{t-j}^2 \tag{20}$$

and

$$diag\{F_t\} \equiv \xi_t = \theta_0 + \theta_1 \odot f_{t-1}^2 + \theta_2 \odot \xi_{t-1}, \tag{21}$$

so that the conditional variance of returns depends upon the conditional variance of the factors. The model is written using vectors of parameters and simple models. The multiplicative model simply replaces (20) with (16). The generalization to an asymmetric component model for the factors and to an asymmetric model for the idiosyncrasies is straightforward.

Maximum likelihood estimation would involve also specifying the process for the correlations among the variables and the covariances with the factors. Instead, the moment conditions associated with estimation of merely the variance equations are considered. This is therefore a GMM or QMLE type of estimation. We will apply a two-step estimation strategy. First the parameters of (21) are estimated. Then (21) is substituted into (20) and the remaining parameters are estimated based on the first step parameters. The quasi-likelihoods for each step are the following

$$QL_1(\theta) = -\tfrac{1}{2} \sum_{t=1}^{T} \sum_{k=1}^{K} [\log(\xi_{k,t}) + f_t^2/\xi_{k,t}], \tag{22}$$

$$QL_2(\Gamma, \omega) = -\tfrac{1}{2} \sum_{t=1}^{T} \sum_{i=1}^{N} [\log(h_{i,t}) + r_{i,t}^2/h_{i,t}]. \tag{23}$$

Since the correlations are not estimated in either case and the joint likelihood is never used, this is a precise example of Newey and McFadden (1994)'s two-step GMM estimator. They present formulae for the standard errors of the two-step estimator but we have not yet implemented these.

The factors can also be extracted directly from the returns data rather than using observable indices. We have applied two approaches: principal components and canonical correlations, and one hybrid which is principal components of a collection of observed sector returns.

The principal components approach is a slight variation of the Orthogonal GARCH model suggested by Alexander (2001) denoted PVCF-PC. An approach close to this is used in the Factor ARCH context by Engle and Ng (1993). The volatilities of the first K principal components of the returns are estimated using the

Component ARCH model of Engle and Lee (1999) where K is the number of variance features.

In the second approach, the variance features are given by the exponential of the first K canonical variates between the logarithmic squared returns and their most recent past. This is the Canonical Correlation PVCF model (PVCF-CC). To motivate this approach, define the squared returns as the variances times the residuals

$$r_t^2 = h_t \odot e_t^2. \tag{24}$$

Taking logs of both sides and adding a very small constant ϖ to deal with exact zeros in recorded returns and approximating the logarithm of the conditional variance in terms of lagged squared returns in logarithms, a logarithmic pth-order ARCH, the equation becomes

$$\begin{aligned} \log(r_t^2 + \varpi) &\doteq \log(h_t) + \log(e_t^2) \\ &\doteq \sum_{j=1}^{p} \alpha_j \log(r_{t-j}^2 + \varpi) + \log(e_t^2). \end{aligned} \tag{25}$$

The canonical correlation procedure seeks linear combinations of the right-hand side variables that are maximally correlated with linear combinations of the left-hand side variables. Thus, the linear combinations of the past squared returns in logarithm, which are highly correlated with their current values, may be a good choice of variance factors. The exponentials of the first K canonical variates are treated as PVCF.

The hybrid approach is the Sector PVCF model (PVCF-SEC), where the variance factors are given by the univariate GARCH volatilities of the largest principal components of the average returns of all economic sectors of the Dow Jones.

Once the PVCF model is estimated, we can employ a set of diagnostic tests for assessing its validity. The first battery of tests involves the portmanteau test for residual autocorrelation in the squared standardized residuals given by Eq. (24). Such tests are equation by equation in spirit and give information about the left-over residual autocorrelation for each single squared return.

Another set of specification tests that can be used is the battery of multivariate tests first introduced by Ding and Engle (2001), in which the orthogonality of the models' residuals is tested. For a well-specified PVCF model, the squared standardized residuals could not be forecast based on any other past information in the model. Ding and Engle (2001) indicate three consequences of correct model specification: (A1) $E(e_t e_t') = I_T$, (A2) $Cov(e_{jt}^2, e_{it}^2) = 0$, for all $i \neq j$, and (A3) $Cov(e_{it}^2, e_{j,t-k}^2) = 0$, for $k > 0$. Because in the PVCF models the correlations between the variables are not jointly modeled, tests of adequacy can only be based on (A3). The null hypothesis in (A3) is equivalent to the moment condition $E(m_t^1) = 0$, where m_t^1 is an N^2 vector with typical element $e_{it}^2 e_{j,t-1}^2$. The empirical moments $\hat{m}_T^1 = T^{-1}\sum_{t=1}^{T} \hat{m}_t^1(\hat{\theta})$ should be close to zero if such condition holds.

Relying on the results of Newey (1985) and Tauchen (1985) for conditional moment testing in a maximum likelihood context, Ding and Engle (2001) suggest several specification tests. These are designed to test whether moment conditions of the form

$$\hat{m}^1_{ij,t} = (\hat{e}^2_{it} - \overline{\hat{e}^2_i})(\hat{e}^2_{j,t-1} - \overline{\hat{e}^2_j}) \tag{26}$$

are satisfied by the model.

Letting $\mathbf{m}^1_t$ be the $T \times k$ matrix of conditional moments, under fairly general conditions we have that $T^{-1/2} i' \mathbf{m}^1_t \xrightarrow{d} N(0, \Omega)$, where i is a $T \times 1$ vector of ones. Therefore, the covariance tests by Ding and Engle (2001) can be viewed as Lagrange Multiplier (LM) tests, whose TR^2_{u} statistics (where R^2_{u} is the uncentered R^2 from the auxiliary regression of ones on the moments) are equivalent to the quadratic form

$$T^{-1} i' \hat{\mathbf{m}}^1_t (\hat{\Omega})^{-1} \hat{\mathbf{m}}^{1\prime}_t i, \tag{27}$$

where $\hat{\Omega} = T^{-1} \hat{\mathbf{m}}^{1\prime}_t \hat{\mathbf{m}}^1_t$ is a consistent estimator of the covariance matrix of the conditional moments. However, since the PVCF models are in a QMLE setting, there remains some theory needed to rigorously establish the distributions of these tests. Actually, the moments might not be martingale difference sequences as a consequence of the dynamic misspecification induced in the standardized residuals $\hat{e}_{it}$ by the use of the conditional variances, instead of the full covariance matrix. This is the reason why we might need a robust estimate of Ω to compute all these tests. We can thus use a non-parametric estimator of the long-run covariance matrix of the empirical moments which is HAC consistent. A natural candidate is the Newey–-West estimator

$$\hat{\hat{\Omega}} = T^{-1} \hat{\mathbf{m}}^{1\prime}_t \hat{\mathbf{m}}^1_t + T^{-1} \sum_{j=1}^{q} w(q) (\hat{\mathbf{m}}^{1\prime}_{t-j} \hat{\mathbf{m}}^1_t + \hat{\mathbf{m}}^{1\prime}_t \hat{\mathbf{m}}^1_{t-j}), \tag{28}$$

where $w(q)$ is the Bartlett kernel and q is a truncation lag. The robust version of the covariance tests is then given by (27) with $\hat{\Omega}$ replaced by (28). This is the version that will be adopted for all the covariance tests presented in this paper.

Ding and Engle (2001) suggest testing all of these moments as (i) the Lagged Covariance test (LC test), which is designed to detect time dependency in multivariate time series, whose test statistic is T times the squared multiple correlation coefficient of the auxiliary regression of constant unity on the empirical moment $\hat{m}^1_T$. The typical element of this set of N^2 moments is defined in (26) as the first lagged sample covariances. The LC test is asymptotically distributed as a $\chi^2_{N^2}$ under the null. The other test is (ii) the Composite Lagged Covariance test (CLC test), whose test statistic is T times the uncentered R^2 from the auxiliary regression of ones on $\hat{M}^1_T$, where the moments are defined as the sum of all the first lagged sample covariances, i.e. $\hat{M}^1_T = \sum_{ij} \hat{m}^1_{ij,t}$. Such test is asymptotically distributed as a χ^2_1 under the null.

A more tractable test is (iii) the Alternative Lagged Covariance test (ALC test), which is designed to detect time dependence of multivariate time series only across assets, thus avoiding redundant ARCH testing within the same asset. The test statistic is similar to the LC test but for the lagged sample covariances included. In the ALC test the $N(N-1)/2$ empirical moments are those in (26) with

$j = i+1, \ldots, N$. Its asymptotic distribution is $\chi^2_{N(N-1)/2}$ under the null. The other two tests are devised to detect possible time dependence at longer lags. They are constructed by adding the moments of the previous tests at each lag. They are: (iv) the Additive Composite Lagged Covariance test at lag k ($ACLC^k$ test) which is calculated using the k moments of each CLC^j test with lag j that goes from 1 to k. (v) The Additive Alternative Lagged Covariance test at lag k ($AALC^k$ test) which is based on the k sums of the ALC test moments at each lag from one to k. The asymptotic distribution of these last two tests under the null is χ^2_k. For all these tests to have the correct distribution under the null of correct specification, they should also include the scores with respect to the estimated parameters. The omission of these additional regressors will only reduce the value of the test statistic, leading it to be conservative.

In the next section, these methods will be examined with the portfolio of the thirty equity returns of the Dow Jones Industrial Average.

5. Modeling common volatilities in the Dow stocks: empirical evidence

5.1. Univariate statistics

The data we analyze in this paper consist of daily returns[2] for the thirty stocks of the Dow Jones Industrial Average Index for a 10-year period from February 20, 1992 to February 20, 2002. For each stock, we have a total of 2609 prices downloaded from Datastream. The Dow Jones Industrial Average is a weighted average of the returns on thirty industrial stocks. The thirty stocks examined in this paper are those that were included in the index in spring 2002. Table 1 gives a complete list of their ticker symbols, company names and the corresponding economic sectors. All these stocks are listed on the New York Stock Exchange, except for Intel and Microsoft that are traded on NASDAQ.

In Table 2, the univariate statistics for the whole data set in percentage terms are presented. The mean for each stock return is on average around 0.05%, while the standard deviation is around 2. Out of the fourteen stock returns considered that show significant skewness, nine exhibit negative skewness (with bigger values for Boeing, Eastman Kodak and Honeywell), while all the others display positive skewness almost close to zero. The kurtosis is always significant and never below 5, thus far away from the normal case of 3. The same conclusion can be easily inferred from the Jarque–Bera test, which rejects the null of normality for all returns at any reasonable significance level. Table 2 also shows the Ljung–Box statistics to test the null hypothesis of absence of serial correlation in both the returns in levels (LB) and

[2]Returns exceeding 20% in absolute value are replaced by the average return over the two most adjacent days. The main reason is that ARCH tests can give low values for the relative statistics, leading to the failure to reject the null hypothesis of no ARCH effects, if the series is characterized by unrepeated 'big events'. With such big jumps in some of the series, we would certainly obtain low values for the common ARCH tests as well, so that more portfolios would misleadingly fail to reject the null hypothesis of absence of ARCH, even though the single asset volatilities moved differently.

Table 1
Ticker symbols, Company names and economic sector of the 30 stocks of the Dow Jones Industrial Average Index

Ticker	Company name	Economic sector
AA	Alcoa	Basic materials
AXP	American Express	Financial
BA	Boeing	Industrial
CAT	Caterpillar	Industrial
CITI[a]	Citigroup	Financial
DIS	Disney	Consumer cyclical
DD	E.I. Du Pont de Nemours	Basic materials
EK	Eastman Kodak	Consumer cyclical
GE	General Electric	Industrial
GM	General Motors	Consumer cyclical
HPQ	Hewlett-Packard	Technology
HD	Home Depot	Consumer cyclical
HON	Honeywell	Industrial
INTC	Intel[b]	Technology
IBM	International Business Machine	Technology
IP	International Paper	Basic materials
JNJ	Johnson & Johnson	Healthcare
JPM	JP Morgan Bank	Financial
KO	Coca Cola	Consumer noncyclical
MCD	McDonalds	Consumer cyclical
MSFT	Microsoft[b]	Technology
MMM	Minnesota Mining and Manufacturing (3M)	Industrial
MO	Philip Morris	Consumer noncyclical
MRK	Merck	Healthcare
PG	Procter and Gamble	Consumer noncyclical
SBC	SBC Communications	Telecommunications
T	AT&T	Telecommunications
UTX	United Technologies	Industrial
WMT	Wal-Mart Stores	Consumer cyclical
XOM	Exxon Mobil	Energy

[a]The original ticker symbol for Citigroup is C but in the paper it is substituted by CITI.
[b]Intel Corporation and Microsoft are quoted in the NASDAQ.

in squares (LB^2) until the 15th lag. The returns in levels show a certain degree of serial correlation, since for twenty out of thirty cases, we reject the null. Furthermore, the LB^2 test on the squared returns indicates the presence of serial correlation at any significance level and, therefore, the existence of ARCH effects. In this case, the theoretical distribution of the LB test is not correct and there is a tendency to over-reject the null.

5.2. Correlation analysis

We also examined the correlation matrix of the thirty returns, both in levels and in squares, to better understand the possible links among different stocks and their

Table 2
Univariate statistics for the Dow Jones daily stock returns

	Mean	Min	Max	Standard deviation	Skewness	Kurtosis	*LB* (15)	LB^2 (15)	Jarque–Bera
AA	0.054	−11.660	13.187	2.083	0.320	6.309	43.26**	307.98**	1234.64**
AXP	0.062	−14.614	11.984	2.107	−0.118	5.889	35.52**	681.43**	913.21**
BA	0.024	−19.389	11.000	1.990	−0.684	12.985	37.69**	141.96**	11041.75**
CAT	0.052	−12.972	10.300	2.051	−0.043	6.014	16.55	101.39**	988.56**
CITI	0.098	−11.496	16.850	2.211	0.206	6.154	25.92*	258.34**	1099.56**
DIS	0.026	−11.701	8.367	1.837	−0.040	5.923	20.20	135.99**	929.23**
DD	0.032	−16.950	14.201	1.996	0.045	8.988	31.18**	218.29**	3898.50**
EK	0.004	−14.363	11.212	1.835	−0.595	10.878	31.75**	61.60**	6900.37**
GE	0.066	−11.287	11.743	1.651	−0.032	7.240	30.49*	461.93**	1954.44**
GM	0.021	−14.540	7.404	1.961	−0.123	5.306	22.65**	117.65**	584.62**
HPQ	0.045	−19.354	19.010	2.674	−0.013	8.056	50.21**	402.33**	2779.53**
HD	0.090	−11.348	12.139	2.132	0.042	5.560	22.12	300.65**	713.16**
HON	0.032	−19.569	11.563	2.108	−0.819	11.978	22.73	89.62**	9053.97**
INTC	0.112	−13.452	18.335	2.768	−0.050	5.983	15.77	70.44**	968.14**
IBM	0.057	−16.889	12.364	2.151	0.024	9.211	33.05**	439.49**	4193.48**
IP	0.004	−11.041	11.242	1.963	0.232	5.868	20.34	514.48**	917.32**
JNJ	0.059	−11.597	7.576	1.607	−0.005	5.342	70.12**	269.80**	596.06**
JPM	0.038	−10.816	14.035	2.157	0.144	5.720	24.38	536.76**	813.40**
KO	0.033	−11.064	9.199	1.660	0.008	6.246	26.54*	460.14**	1145.54**
MCD	0.036	−11.093	10.322	1.676	0.062	6.501	17.08	211.27**	1334.33**
MSFT	0.096	−16.953	17.877	2.317	−0.110	7.612	29.12*	140.22**	2317.87**
MMM	0.037	−10.078	10.505	1.559	0.097	6.721	47.92**	208.42**	1509.31**
MO	0.038	−14.938	15.061	1.970	−0.091	9.830	27.01*	106.40**	5075.24**
MRK	0.035	−9.860	9.161	1.779	−0.030	5.480	33.86**	216.84**	668.80**
PG	0.060	−10.238	9.091	1.667	−0.209	7.003	30.23*	399.00**	1760.88**
SBC	0.035	−13.538	8.845	1.795	−0.085	6.307	18.65	354.02**	1192.33**
T	0.005	−14.890	12.399	2.058	0.085	9.170	18.07	468.56**	4141.17**
UTX	0.078	−13.482	7.730	1.737	−0.134	5.889	33.85**	435.63**	915.23**
WMT	0.058	−10.268	9.015	2.036	0.088	5.293	52.94**	458.71**	574.91**
XOM	0.038	−7.664	10.481	1.408	0.207	5.576	58.36**	250.81**	740.04**

Note: The sample is 2/20/1992–2/20/2002. The descriptive statistics are calculated on the daily percentage returns. The skewness, the kurtosis and the Jarque–Bera test are calculated on the standardized returns to make results comparable. *LB*(15) and LB^2(15) are the Ljung–Box statistics to test the null of absence of serial correlation in the residuals and in their squares, respectively, up to the 15th lag. * and ** indicate significance at 5% and 1%, respectively.

volatilities. Table 3 shows the correlation matrix for the returns in levels in the lower-left triangle and for the squares in the upper right triangle.

The stock returns do exhibit positive and significant correlations with each other not only in the levels, but also in the squares. Out of the thirty correlation coefficients in the levels, seven are even higher than 0.50 and, among these, one is bigger than 0.60. Not surprisingly, the strongest correlations are between stocks within the same industry: American Express and Citigroup, American Express and JP Morgan, JP Morgan and Citigroup, Wal-Mart Stores and Home Depot, Microsoft and Intel, Johnson & Johnson and Merck. However, there is also a very strong correlation

Table 3
Correlation matrix for returns (lower left triangle) and squared returns (upper right triangle)

	Financial			Consumer cyclical						Consumer noncyclical			Healthcare		Technology				Telecomm.		Energy	Basic materials			Industrial					
	AXP	CITI	JPM	EK	GM	HD	MCD	WMT	DIS	KO	MO	PG	JNJ	MRK	HPQ	INTC	IBM	MSFT	T	SBC	XOM	IP	AA	DD	BA	CAT	GE	HON	MMM	UTX
AXP	1	0.38	**0.45**	0.09	0.39	**0.43**	0.22	0.30	0.20	0.23	0.08	0.21	0.13	0.19	0.15	0.25	0.14	0.21	0.18	0.21	0.15	0.32	0.27	0.28	0.36	0.18	**0.49**	0.37	0.26	0.24
CITI	**0.55**	1	**0.47**	0.07	0.21	0.29	0.19	0.22	0.15	0.21	0.07	0.21	0.16	0.18	0.11	0.19	0.14	0.15	0.15	0.14	0.18	0.20	0.17	0.20	0.18	0.15	0.32	0.20	0.19	0.24
JPM	**0.55**	**0.61**	1	0.10	0.23	0.36	0.16	0.24	0.23	0.21	0.13	0.21	0.12	0.16	0.18	0.24	0.18	0.23	0.18	0.17	0.15	0.18	0.15	0.15	0.12	0.14	**0.40**	0.20	0.19	0.26
EK	0.22	0.23	0.24	1	0.12	0.12	0.05	0.03	0.08	0.04	0.03	0.06	0.03	0.05	0.05	0.10	0.05	0.11	0.08	0.05	0.08	0.11	0.12	0.10	0.11	0.11	0.13	0.14	0.08	0.22
GM	0.31	0.34	0.35	0.20	1	0.34	0.11	0.19	0.10	0.13	0.03	0.12	0.08	0.12	0.13	0.20	0.05	0.16	0.10	0.09	0.10	0.28	0.24	0.28	0.37	0.16	0.39	0.36	0.20	0.20
HD	0.39	**0.41**	0.39	0.19	0.30	1	0.26	**0.48**	0.25	0.24	0.09	0.24	0.16	0.24	0.22	0.27	0.17	0.26	0.19	0.14	0.16	0.28	0.23	0.23	0.28	0.18	**0.47**	0.28	0.25	0.28
MCD	0.24	0.24	0.22	0.15	0.18	0.28	1	0.29	0.16	0.25	0.05	0.18	0.18	0.21	0.05	0.09	0.13	0.18	0.14	0.10	0.15	0.16	0.13	0.12	0.16	0.19	0.21	0.09	0.20	0.23
WMT	0.34	0.34	0.32	0.16	0.24	**0.54**	0.28	1	0.15	0.29	0.09	0.23	0.20	0.26	0.10	0.20	0.16	0.17	0.20	0.16	0.19	0.17	0.13	0.16	0.16	0.16	0.31	0.13	0.19	0.22
DIS	0.28	0.29	0.29	0.18	0.20	0.28	0.19	0.23	1	0.18	0.08	0.13	0.09	0.14	0.12	0.13	0.10	0.12	0.17	0.08	0.10	0.09	0.08	0.07	0.11	0.12	0.21	0.11	0.08	0.19
KO	0.27	0.27	0.23	0.16	0.14	0.25	0.28	0.28	0.18	1	0.11	0.23	0.20	0.26	0.06	0.10	0.11	0.12	0.21	0.14	0.15	0.13	0.06	0.08	0.14	0.13	0.23	0.10	0.17	0.26
MO	0.16	0.19	0.14	0.14	0.10	0.14	0.17	0.12	0.15	0.22	1	0.08	0.10	0.09	0.04	0.05	0.17	0.09	0.09	0.08	0.11	0.15	0.05	0.07	0.04	0.04	0.12	0.06	0.04	0.07
PG	0.24	0.26	0.20	0.15	0.15	0.22	0.30	0.26	0.14	**0.40**	0.23	1	0.20	0.21	0.09	0.14	0.12	0.12	0.21	0.19	0.18	0.18	0.10	0.18	0.12	0.14	0.22	0.12	0.17	0.26
JNJ	0.24	0.26	0.19	0.13	0.13	0.22	0.24	0.25	0.16	0.32	0.21	0.35	1	0.30	0.05	0.09	0.11	0.09	0.12	0.07	0.16	0.15	0.09	0.07	0.08	0.11	0.15	0.11	0.13	0.18
MRK	0.22	0.25	0.23	0.13	0.18	0.23	0.22	0.25	0.15	0.30	0.23	0.33	**0.53**	1	0.07	0.08	0.12	0.15	0.16	0.10	0.18	0.20	0.14	0.16	0.14	0.16	0.18	0.12	0.15	0.19
HPQ	0.24	0.26	0.29	0.19	0.22	0.28	0.16	0.19	0.24	0.12	0.10	0.09	0.10	0.12	1	0.24	0.17	0.14	0.10	0.05	0.06	0.12	0.10	0.10	0.12	0.09	0.20	0.15	0.10	0.13
INTC	0.29	0.31	0.32	0.19	0.27	0.29	0.15	0.21	0.21	0.12	0.09	0.12	0.13	0.13	**0.47**	1	0.24	0.31	0.15	0.09	0.11	0.17	0.15	0.14	0.17	0.14	0.28	0.22	0.18	0.18
IBM	0.26	0.30	0.28	0.15	0.21	0.24	0.17	0.20	0.20	0.11	0.13	0.12	0.14	0.15	**0.41**	**0.40**	1	0.14	0.13	0.05	0.10	0.21	0.09	0.11	0.09	0.14	0.16	0.13	0.07	0.12
MSFT	0.28	0.33	0.31	0.19	0.25	0.31	0.14	0.25	0.23	0.16	0.13	0.13	0.18	0.19	0.39	**0.55**	0.35	1	0.16	0.10	0.08	0.14	0.13	0.17	0.15	0.13	0.24	0.15	0.16	0.12
T	0.23	0.27	0.22	0.13	0.23	0.24	0.14	0.20	0.22	0.13	0.13	0.12	0.12	0.15	0.22	0.23	0.20	0.24	1	0.19	0.15	0.16	0.11	0.12	0.11	0.08	0.21	0.12	0.09	0.16
SBC	0.24	0.23	0.23	0.13	0.13	0.17	0.21	0.22	0.14	0.24	0.17	0.24	0.22	0.18	0.09	0.11	0.12	0.14	0.27	1	0.10	0.15	0.11	0.13	0.04	0.08	0.13	0.07	0.11	0.16
XOM	0.18	0.21	0.16	0.13	0.10	0.17	0.15	0.18	0.15	0.26	0.17	0.21	0.20	0.18	0.09	0.10	0.12	0.15	0.13	0.21	1	0.17	0.17	0.23	0.15	0.18	0.25	0.17	0.19	0.15
IP	0.26	0.24	0.25	0.22	0.28	0.27	0.16	0.20	0.18	0.14	0.13	0.14	0.11	0.13	0.13	0.14	0.12	0.16	0.16	0.11	0.15	1	**0.49**	0.37	0.28	0.27	0.31	0.32	0.21	0.23
AA	0.25	0.27	0.26	0.21	0.25	0.25	0.17	0.18	0.18	0.12	0.12	0.13	0.10	0.11	0.17	0.16	0.17	0.16	0.15	0.12	0.20	**0.45**	1	0.35	0.27	0.32	0.27	0.28	0.17	0.19
DD	0.29	0.30	0.26	0.24	0.30	0.26	0.19	0.26	0.17	0.23	0.17	0.24	0.16	0.19	0.17	0.15	0.16	0.14	0.14	0.15	0.23	**0.43**	0.39	1	0.30	0.24	0.30	0.32	0.29	0.17
BA	0.26	0.26	0.24	0.17	0.21	0.28	0.16	0.22	0.19	0.19	0.10	0.16	0.17	0.17	0.17	0.20	0.16	0.19	0.14	0.12	0.18	0.22	0.23	0.26	1	0.19	0.39	**0.43**	0.21	0.24
CAT	0.29	0.31	0.27	0.23	0.30	0.30	0.21	0.24	0.19	0.19	0.16	0.18	0.14	0.14	0.19	0.18	0.19	0.20	0.14	0.13	0.17	0.39	0.37	0.38	0.25	1	0.21	0.18	0.17	0.22
GE	**0.50**	**0.49**	**0.46**	0.23	0.34	**0.44**	0.30	**0.41**	0.35	0.34	0.18	0.32	0.31	0.28	0.30	0.32	0.32	0.35	0.28	0.23	0.24	0.31	0.30	0.35	0.32	0.33	1	**0.48**	0.29	0.32
HON	0.33	0.36	0.32	0.23	0.27	0.29	0.18	0.23	0.23	0.21	0.15	0.17	0.14	0.16	0.23	0.22	0.22	0.25	0.19	0.19	0.22	0.33	0.32	0.35	0.33	0.34	**0.42**	1	0.21	0.26
MMM	0.30	0.30	0.28	0.24	0.27	0.26	0.18	0.26	0.17	0.24	0.18	0.24	0.19	0.20	0.17	0.18	0.17	0.14	0.15	0.17	0.23	0.37	0.28	**0.43**	0.26	0.33	0.35	0.32	1	0.22
UTX	0.32	0.35	0.32	0.22	0.27	0.30	0.23	0.26	0.22	0.24	0.13	0.24	0.18	0.19	0.21	0.17	0.20	0.22	0.14	0.18	0.22	0.29	0.30	0.32	0.34	0.34	0.39	0.39	0.34	1

Note: The table shows the correlation matrix for the percentage returns in levels (lower left triangle) and for the squared returns (upper right triangle). The numbers in boldface represent the correlation coeffecients greater than or equal to 0.40.

between General Electric and American Express which are in different economic sectors, although the former does have important financial business so that this may not be so surprising.

The correlations between squared returns are naturally related to the correlations between the levels of returns, but can be helpful to discover possible comovements in their volatilities. From the upper triangle in Table 2, we can see that there are seven correlation coefficients above 0.45 and almost all correspond to stocks within the same business area: American Express and JP Morgan, JP Morgan and Citigroup, Home Depot and Wal-Mart, International Paper and Alcoa, Honeywell and General Electric. The other strong correlations outside the same economic sector are those between General Electric and American Express, and General Electric and Home Depot. Other correlations (Home Depot and American Express, Honeywell and Boeing) are quite strong. All these results indicate that there are strong comovements in the volatilities of the thirty stocks in the Dow Jones. Most of these comovements are within or between specific industries, showing the possible presence of more than just one volatility factor. In particular, we can infer the existence of industry-specific volatility factors together with a global or market factor.[3]

5.3. ARCH tests and univariate GARCH estimates

Table 4 reports some ARCH tests. These are LM tests with univariate information sets. Each return is squared and used as a proxy for the realized volatility. Then, each squared return is regressed on a constant and 4, 8 and 12 lags of the same squared return. The statistic is obtained by multiplying the uncentered R^2 from this regression by the sample size, and asymptotically is distributed as a χ^2_p where p is the number of lags.

We thus find strong evidence of ARCH effects at any lag, for all stocks. These results are corroborated by the univariate GARCH estimates for each stock return, which are highly significant for all the thirty assets examined and are not reported for the sake of brevity.

Fig. 1 displays annualized volatilities for nine of the thirty assets, as estimated from individual univariate GARCH. We report the annualized volatilities for only few stocks, because others resemble volatility patterns that are similar to the ones presented. Nevertheless, the volatility patterns seem to be quite different among the thirty assets of the Dow, indicating that there might be more than one volatility factor which drives all these volatilities.

5.4. Testing for common ARCH features

Since all stock returns in the Dow Jones show ARCH effects, we test for the presence of common ARCH features, following the Engle and Susmel's (1993) pairwise methodology. Whenever two series display ARCH effects, we test for

[3]These correlation results do not change if we adopt a sort of preprocessing of the returns, such as fitting individual AR processes or a VAR.

Tabel 4
ARCH tests for r_t

	LAGS		
	4	8	12
AA	61.49*	97.96*	144.11*
AXP	232.34*	325.91*	331.34*
BA	101.11*	113.90*	118.99*
CAT	42.30*	54.05*	74.85*
CITI	84.69*	122.39*	137.15*
DIS	52.11*	65.85*	86.50*
DD	57.45*	84.44*	104.08*
EK	49.91*	50.11*	51.41*
GE	155.86*	255.68*	261.12*
GM	45.73*	69.17*	78.41*
HPQ	31.76*	44.23*	51.95*
HD	112.56*	154.23*	185.05*
HON	159.23*	186.99*	193.58*
INTC	161.23*	185.21*	202.81*
IBM	26.00*	35.09*	44.20*
IP	144.15*	194.93*	214.57*
JNJ	51.27*	109.28*	131.02*
JPM	125.05*	191.88*	208.04*
KO	175.62*	187.79*	208.84*
MCD	109.02*	181.66*	187.92*
MSFT	45.86*	88.58*	122.23*
MMM	52.73*	82.05*	91.41*
MO	88.93*	107.42*	127.66*
MRK	41.74*	57.61*	75.81*
PG	154.12*	204.44*	217.37*
SBC	87.28*	126.46*	161.94*
T	164.95*	187.35*	210.52*
UTX	160.64*	180.37*	197.34*
WMT	82.90*	151.59*	195.65*
XOM	93.86*	116.72*	126.67*

*Significance at the 5%.

common ARCH features (or factor ARCH), by seeking those linear combinations for which this feature is not present. In fact, we look for those portfolio weights for which the variance of the whole portfolio only depends on the volatilities of the idiosyncrasies. The test is implemented by minimizing the usual LM ARCH test over the cofeature vector. If x_t and y_t have both ARCH effects, we minimize the ARCH test for $w_t = x_t - \delta y_t$ with respect to δ. The procedure involves a regression of w_t^2 against lagged squared values of both series and their cross products, followed by the minimization of TR^2 over the parameter δ. This is a general method-of-moment-type of test and under suitable assumptions it follows a χ^2 distribution with degrees of freedom equal to the number of overidentifying restrictions (see Engle and Kozicki, 1993). Engle and Susmel (1993) look for the minimum by performing a grid search.

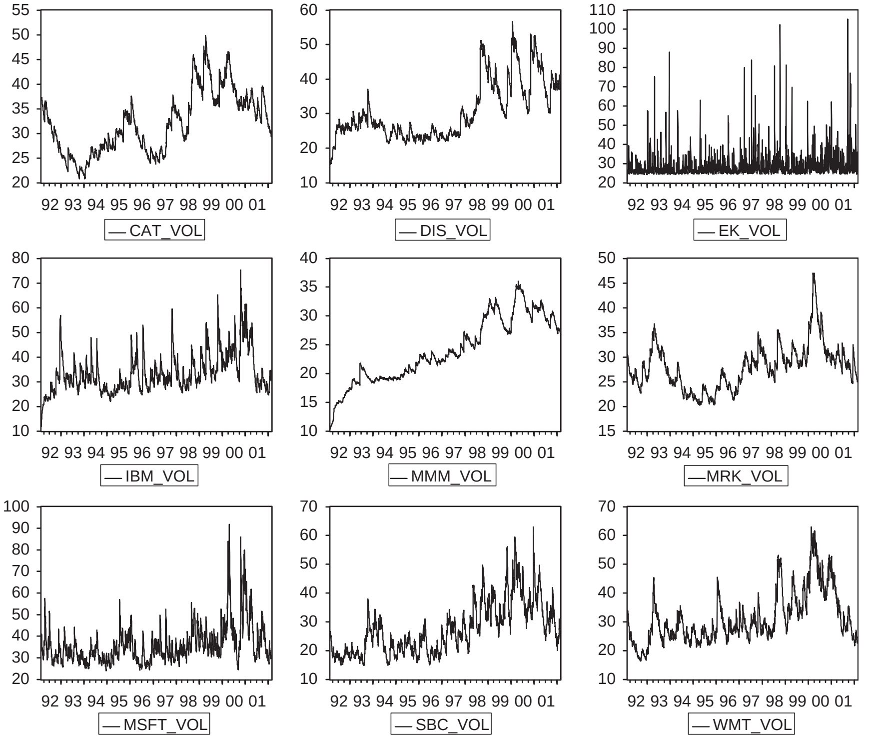

Fig. 1. GARCH volatilities for nine of the assets in the Dow Jones. *Notes:* The figure depicts the annualized GARCH(1,1) volatilities for returns of Caterpillar, Disney, Eastman Kodak, IBM, Minnesota Mining and Manufacturing, Merck, Microsoft, SBC Communications, and Wal-Mart Stores.

In the present paper, we use both a grid search and the BFGS (Broyden–Fletcher–Goldfarb–Shanno) method,[4] but the final results do not change considerably.

We carry out the test for common ARCH features on the squared returns and out of the 435 possible two-asset portfolios, only four pairs show a common ARCH feature, at the usual 5% significance level. We also perform other kinds of testing along the same line. We minimize the ARCH LM test for combinations of various net returns, by subtracting from each series the return on some market indices, such as S&P500 and NASDAQ. We adopt the same testing procedure by including in the portfolios either the S&P500 or the NASDAQ, and by looking for common ARCH features among three-asset portfolios. The main conclusion is that the preceding results still hold: only very few portfolios show a common ARCH feature.

This finding is not completely new in the literature: Alexander (1995) finds no-ARCH portfolio for any exchange rate pairs, using daily data from a 10-year sample of dollar returns on seven currencies and from some of its sub-samples. Her main

[4]The BFGS is a quasi-Newton optimization method that does not imply the calculation of the Hessian matrix of the objective function.

conclusions are twofold: firstly daily data contain so much noise that it is really hard to find a common feature. Secondly, the Engle–Kozicki test might have reduced power in a dynamic setting. This latter line of argument follows Ericsson's (1993) critique to Engle and Kozicki (1993). Ericsson argues that, in a bivariate setting, the cofeature hypothesis might be too restrictive, and, consequently, it could be rejected, even if the cofeature does exist. To solve this problem, it would be sufficient using multivariate procedures, in such a way to include in the information set all the lagged data and not only the lags for the pair of variables under investigation.

As a matter of fact, we select two sub-samples from our data set, and run the same common factor ARCH tests. Again and not surprisingly, very few pairs fail to reject the null of no ARCH effects, leading to the rejection of a common ARCH feature.

When the same test is run on the logarithmic transformation of squared returns plus a tiny constant to make the transformed series closer to being normally distributed, the results turn out to be much more encouraging,[5] because more evidence of CF is found. Actually, almost all the portfolios exhibit a common ARCH feature, since only for 35 pairs out of 435 the null of no ARCH effects is rejected. The interpretation of such CF is however different.

A further explanation for the difficulties in finding common ARCH factors is closely related to the assumed factor structure. As Engle and Susmel (1993) point out, if the idiosyncratic components in the model do not have a constant covariance matrix, there will be no portfolio that shows a constant variance–covariance matrix, because even though one can find a coefeature vector that annihilates the matrix of factor loadings, there will still be a time-varying volatility component, due to the idiosyncrasies, that cannot be diversified.

Thus, in the next section we will look for possible LRPVCF, taking into account the fact that the idiosyncrasies seem to exhibit ARCH-like time-varying volatilities. This fact implies the presence of variance features that are common to more than just a pair of asset returns and calls for a more general search that must necessarily be multivariate.

5.5. How many Pure Variance Common Features are in the Dow?

Fig. 1 illustrates how individual GARCH annualized volatilities seem to display very different patterns. This means that there is more than one pure variance factor driving the whole volatility process of the Dow Jones Industrial Index. As argued before, one variance factor can be related to the market, but there is evidence of the possible presence of industry-specific variance factors.

The problem of finding a cofeature vector can be analyzed in a reduced rank regression framework, by means of canonical correlation analysis (see Anderson,

[5]For brevity we do not report the corresponding tables that are available upon request from the authors.

1984, 1999). The number of pure variance factors is equivalent to the rank of the matrix Π in

$$r_t^2 \doteq \sum_{j=1}^{J} \pi_j \odot r_{t-j}^2 + \eta_t = \Pi \Upsilon_t + \eta_t, \quad \Pi = (\pi_1' \quad \cdots \quad \pi_J'),$$

$$\Upsilon_t = \left((r_{t-1}^2)' \quad \cdots \quad (r_{t-J}^2)' \right)' \tag{29}$$

which could be evaluated by means of the Bartlett test for the significance of the smallest canonical correlation coefficients if squared returns were normally distributed.

The main drawback in using canonical correlation analysis is the fundamental assumption of normality on which all multivariate techniques are based. The squared returns are highly non-normal and applying the canonical correlation analysis to them can lead to an overestimation of the number of pure variance factors. In the multivariate statistical literature many corrections have been proposed so far, but the majority only considers elliptical distributions with possible non-zero kurtosis and finite fourth moments. Gunderson and Muirhead (1997) and Yuan and Bentler (2000) suggest some of these tests, building on the results of Muirhead (1982) and Muirhead and Waternaux (1980).

To overcome this problem, we adopt the logarithmic transformation that is typically used in the stochastic volatility literature. We add a tiny constant to each squared return and then take the logarithm. The tiny constant is chosen by minimizing the distances between the skewness and the kurtosis of the transformed data and their normal values.

Therefore, we calculate the canonical correlations of $y_t = \log(r_t^2 + \varpi)$ with respect to $w_{t,p} = (y_{t-1}, \ldots, y_{t-p})$, that is the matrix of lagged transformed squared returns. We choose five lags for the log-transformed squared returns on the right-hand side, in order to find the variance factors related to the last five business days. As noted by Box and Tiao (1977), the largest canonical correlations will correspond to the most predictable log-transformed squared returns with respect to their most recent past.

Table 5 illustrates the results from canonical correlation analysis on the squared returns under the logarithmic and the Box–Cox transformation. We report the estimated canonical correlations and two tests (the Bartlett and the Bartlett–Lawley tests) for the significance of the first $k-1$ canonical correlation coefficients, $k = 1,\ldots,p$, with the corresponding degrees of freedom and p-values. The null hypothesis is equivalent to $rank(\Gamma) = k-1$, where $\Gamma = \Sigma_{22}^{-1}\Sigma_{21}$ is the matrix of regression coefficients of y_t on $w_{t,p}$.

There is no practical difference between the two sequential tests, and the number of pure variance factors found is two for the logarithmic transformation and three for the Box–Cox. If more lags are taken for the right-hand side variables $w_{t,p}$, only the first canonical correlation remains significant for the log-transformed squared returns, while the previous results still hold for the Box–Cox transformation.

Table 5
Canonical correlation analysis of the transformed squared returns with respect to their most recent past

	Logarithmic transformation: $y_{1t} = \log(r_t^2 + \varpi)$						Box–Cox transformation: $y_{2t} = \left[(r_t^2)^{\lambda} - 1\right]/\lambda$					
k	CC	Bartlett LR test	p-value	Bartlett–Lawley LR test	p-value	DoF	CC	Bartlett LR test	p-value	Bartlett–Lawley LR test	p-value	DoF
0	0.423*	5027.01	0.00	5038.22	0.00	4500	0.468*	5356.01	0.00	5365.75	0.00	4500
1	0.346*	4531.08	0.01	4556.23	0.01	4321	0.364*	4733.64	0.00	4756.48	0.00	4321
2	0.337	4208.53	0.24	4246.63	0.13	4144	0.344*	4374.94	0.01	4410.81	0.00	4144
3	0.327	3903.60	0.77	3953.49	0.57	3969	0.330	4057.81	0.16	4105.94	0.06	3969
4	0.317	3618.23	0.98	3678.83	0.91	3796	0.321	3767.00	0.63	3826.29	0.36	3796
5	0.294	3350.96	1.00	3422.56	0.99	3625	0.303	3493.10	0.94	3563.30	0.76	3625
6	0.291	3122.85	1.00	3204.34	1.00	3456	0.295	3250.78	0.99	3331.06	0.93	3456
7	0.282	2900.51	1.00	2990.79	1.00	3289	0.290	3021.68	1.00	3110.71	0.99	3289
8	0.277	2691.74	1.00	2789.52	1.00	3124	0.284	2801.14	1.00	2897.53	1.00	3124
9	0.270	2490.27	1.00	2594.36	1.00	2961	0.279	2588.72	1.00	2691.15	1.00	2961
10	0.263	2299.15	1.00	2408.54	1.00	2800	0.271	2385.41	1.00	2492.83	1.00	2800
11	0.255	2118.37	1.00	2232.18	1.00	2641	0.264	2193.97	1.00	2305.38	1.00	2641
12	0.254	1948.83	1.00	2065.68	1.00	2484	0.258	2012.13	1.00	2126.43	1.00	2484
13	0.250	1781.80	1.00	1900.14	1.00	2329	0.254	1839.09	1.00	1954.97	1.00	2329
14	0.244	1620.49	1.00	1739.08	1.00	2176	0.252	1671.21	1.00	1787.10	1.00	2176
15	0.235	1467.13	1.00	1585.18	1.00	2025	0.242	1506.76	1.00	1621.58	1.00	2025
16	0.231	1324.72	1.00	1441.32	1.00	1876	0.238	1355.53	1.00	1468.49	1.00	1876
17	0.229	1187.54	1.00	1301.14	1.00	1729	0.226	1209.94	1.00	1320.27	1.00	1729
18	0.217	1052.13	1.00	1161.76	1.00	1584	0.222	1078.23	1.00	1185.34	1.00	1584
19	0.213	931.14	1.00	1036.42	1.00	1441	0.216	951.63	1.00	1054.38	1.00	1441
20	0.211	814.83	1.00	914.31	1.00	1300	0.213	832.00	1.00	929.21	1.00	1300
21	0.202	700.60	1.00	793.08	1.00	1161	0.202	715.66	1.00	806.38	1.00	1161
22	0.194	596.90	1.00	682.06	1.00	1024	0.198	612.10	1.00	696.00	1.00	1024
23	0.189	500.66	1.00	577.76	1.00	889	0.189	512.44	1.00	588.45	1.00	889
24	0.182	410.26	1.00	478.45	1.00	756	0.185	421.38	1.00	488.84	1.00	756
25	0.171	326.29	1.00	385.04	1.00	625	0.182	334.33	1.00	391.91	1.00	625
26	0.167	252.55	1.00	301.68	1.00	496	0.172	250.02	1.00	296.51	1.00	496
27	0.164	182.16	1.00	220.34	1.00	369	0.164	175.60	1.00	210.88	1.00	369
28	0.153	114.62	1.00	140.63	1.00	244	0.150	107.50	1.00	131.04	1.00	244
29	0.149	55.56	1.00	69.18	1.00	121	0.143	51.07	1.00	63.27	1.00	121

Note: The table shows the Canonical Correlation Analysis performed between y_{jt} as the LHS variables and $w_{jt,q} = \{y_{j,t-1}, \ldots, y_{j,t-q}\}$ with $j = 1, 2$, and $q = 5$ as the RHS variables. CC are the sample canonical correlations, the null hypothesis for both LR tests is that the last $p - k$ canonical correlations are zero against the alternative that $CC_j \neq 0$ for some $j \geq k + 1$. * indicates the canonical correlations that are significant at 5% significance level.

5.6. Estimating the long-run Pure Variance Common Feature model for the Dow

If we consider the additive factor structure, we can adopt either an ARCH(p) as in (14) or a GJR(p,0) for the short-run volatility component. In the latter case, the conditional variance of each return becomes

$$h_{it} = \alpha_{i0} + \sum_{j=1}^{p} \alpha_{ij} r_{i,t-j}^2 + \gamma r_{i,t-1}^2 I_{\{r_{i,t-1}>0\}} + \sum_{k=1}^{K} \gamma_{ik} \xi_{kt}, \tag{30}$$

where $I_{\{r_{i,t-1}>0\}}$ is 1 if $r_{i,t-1}>0$ and 0 otherwise, and ξ_{kt} is the kth pure variance common factor. If we work with the corresponding multiplicative factor structure, we can adopt an EGARCH(p,0) framework as in (16) for the idiosyncrasies.

The latent volatility factors represent commonalities in the conditional variances that drive the comovements in the asset return volatilities, in response to different news. Such factors can be specified using both statistical and observable approaches. The former involves a factor extraction from the returns by means of a particular statistical method. The latter implies the specification of the factors based on the argument that they capture economy-wide systematic risk.

In the observable factor approach, we use the GARCH volatilities of the major US stock indices, such as the Dow Jones itself, the NASDAQ, the S&P500 and the S&P100. We also employ the GARCH volatilities of the first three principal components of the average returns of all the sectors in the Dow. Actually, we think that most of the commonalities in the Dow Jones volatilities are linked not only to these key US stock market indices, but also to the different economic sectors of the Dow.

All PVCF models are estimated by quasi-maximum likelihood, treating the factors estimated in a first step as given. For each model we calculate the principal components of the estimated conditional volatilities. This is a descriptive statistic designed to measure how closely the volatilities move together. For example, in a one-factor PVCF model with constant idiosyncratic volatility, the first principal component would explain all the variability, since all volatilities would move proportionally to each other. With increased idiosyncratic volatility, the proportion of all volatility explained by the first factor would decline. Other factors would also contribute, so the explanatory proportion due to the second and third principal component would reveal the importance of additional factors. Furthermore, the Ljung–Box test for the squared standardized residuals up to the 15th lag is evaluated to assess how much serial correlation (and hence ARCH effect) is left on each residual. For each PVCF model five covariance tests are also computed to assess temporal dependence left in the residuals of the whole multivariate model.

Tables 6 and 7 give a summary of these tests for all the PVCF models estimated. For each model, the tables show the cumulative variance explained by the first four principal components, the covariance tests that each model passes, and the number of assets (out of thirty) for which we fail to reject the null of absence of serial correlation in the squared standardized residuals until the 15th lag. The tables exhibit these results for the models with statistical factors (PVCF-PC and PVCF-CC)

Table 6
Comparison of the PVCF models with factors from principal components, canonical correlations and economic sectors

Model	1pc	2pc	3pc	4pc	Covariance tests	LB^2	Model	1pc	2pc	3pc	4pc	Covariance tests	LB^2	Model	1pc	2pc	3pc	4pc	Covariance tests	LB^2
GARCH(1,0)-1PC	0.85	0.87	0.89	0.91	—	11	GARCH(1,0)-1CC	0.87	0.90	0.91	0.92	—	14	GARCH(1,0)-1SEC	0.78	0.81	0.84	0.85	—	10
GARCH(1,0)-2PC	0.83	0.87	0.90	0.92	v	12	GARCH(1,0)-2CC	0.74	0.80	0.83	0.85	—	10	GARCH(1,0)-2SEC	0.80	0.86	0.88	0.90	vi	12
GARCH(1,0)-3PC	0.82	0.87	0.90	0.92	i, iii, v	15	GARCH(1,0)-3CC	0.46	0.54	0.59	0.63	—	7	GARCH(1,0)-3SEC	0.79	0.85	0.88	0.89	vi	13
GARCH(3,0)-1PC	0.80	0.82	0.84	0.86	—	22	GARCH(3,0)-1CC	0.81	0.84	0.86	0.87	ii	22	GARCH(3,0)-1SEC	0.71	0.74	0.76	0.78	—	16
GARCH(3,0)-2PC	0.79	0.84	0.86	0.88	ii, iii, v	23	GARCH(3,0)-2CC	0.63	0.70	0.73	0.76	—	15	GARCH(3,0)-2SEC	0.75	0.81	0.83	0.85	ii, v	23
GARCH(3,0)-3PC	0.79	0.83	0.87	0.89	i, iii, v	23	GARCH(3,0)-3CC	0.43	0.53	0.58	0.62	—	11	GARCH(3,0)-3SEC	0.75	0.80	0.83	0.85	ii, iv ,v	24
GARCH(5,0)-1PC	0.73	0.76	0.79	0.81	—	26	GARCH(5,0)-1CC	0.76	0.79	0.81	0.83	ii	22	GARCH(5,0)-1SEC	0.66	0.70	0.73	0.75	ii	26
GARCH(5,0)-2PC	0.75	0.79	0.81	0.83	ii, iv,v	26	GARCH(5,0)-2CC	0.64	0.69	0.72	0.75	—	20	GARCH(5,0)-2SEC	0.69	0.75	0.78	0.80	ii, v	26
GARCH(5,0)-3PC	0.75	0.79	0.82	0.85	i, iii, v	25	GARCH(5,0)-3CC	0.43	0.51	0.56	0.60	—	11	GARCH(5,0)-3SEC	0.69	0.75	0.77	0.79	ii, v	26
EGARCH(1,0)-1PC	0.85	0.89	0.91	0.92	—	10	EGARCH(1,0)-1CC	0.85	0.87	0.89	0.90	—	9	EGARCH(1,0)-1SEC	0.83	0.86	0.88	0.90	—	9
EGARCH(1,0)-2PC	0.84	0.88	0.91	0.93	v	9	EGARCH(1,0)-2CC	0.79	0.86	0.89	0.90	—	11	EGARCH(1,0)-2SEC	0.83	0.87	0.90	0.92	—	10
EGARCH(1,0)-3PC	0.83	0.87	0.90	0.93	i, iii, v	15	EGARCH(1,0)-3CC	0.76	0.84	0.88	0.90	—	12	EGARCH(1,0)-3SEC	0.82	0.87	0.90	0.92	vi	12
EGARCH(3,0)-1PC	0.77	0.80	0.82	0.85	—	20	EGARCH(3,0)-1CC	0.75	0.78	0.80	0.82	—	17	EGARCH(3,0)-1SEC	0.74	0.77	0.79	0.81	—	14
EGARCH(3,0)-2PC	0.78	0.81	0.84	0.86	ii, iii, v	23	EGARCH(3,0)-2CC	0.71	0.78	0.81	0.83	—	17	EGARCH(3,0)-2SEC	0.75	0.80	0.83	0.85	ii, v	20
EGARCH(3,0)-3PC	0.78	0.81	0.85	0.87	i, iii, v	24	EGARCH(3,0)-3CC	0.69	0.77	0.81	0.83	—	19	EGARCH(3,0)-3SEC	0.75	0.80	0.82	0.85	ii, v	21
EGARCH(5,0)-1PC	0.73	0.76	0.79	0.81	—	25	EGARCH(5,0)-1CC	0.71	0.74	0.76	0.78	—	21	EGARCH(5,0)-1SEC	0.69	0.72	0.75	0.77	—	22
EGARCH(5,0)-2PC	0.75	0.78	0.81	0.83	ii, iii, v	25	EGARCH(5,0)-2CC	0.67	0.74	0.77	0.79	—	22	EGARCH(5,0)-2SEC	0.71	0.76	0.79	0.81	ii, v	25
EGARCH(5,0)-3PC	0.75	0.78	0.82	0.84	i, iii, v	26	EGARCH(5,0)-3CC	0.66	0.73	0.78	0.80	—	22	EGARCH(5,0)-3SEC	0.71	0.76	0.78	0.81	ii, v	25
GJR(1,0)-1PC	0.82	0.84	0.87	0.89	—	13	GJR(1,0)-1CC	0.85	0.88	0.89	0.90	—	14	GJR(1,0)-1SEC	0.73	0.77	0.80	0.82	—	10
GJR(1,0)-2PC	0.79	0.83	0.86	0.88	v	12	GJR(1,0)-2CC	0.73	0.78	0.81	0.83	—	9	GJR(1,0)-2SEC	0.75	0.81	0.84	0.87	vi	12
GJR(1,0)-3PC	0.79	0.83	0.87	0.89	i, iii, v	14	GJR(1,0)-3CC	0.47	0.55	0.59	0.63	—	8	GJR(1,0)-3SEC	0.76	0.82	0.84	0.87	vi	14
GJR(3,0)-1PC	0.77	0.79	0.82	0.84	ii	22	GJR(3,0)-1CC	0.80	0.83	0.85	0.86	ii	22	GJR(3,0)-1SEC	0.67	0.71	0.74	0.76	vi	17
GJR(3,0)-2PC	0.76	0.81	0.83	0.86	ii, iii, v	24	GJR(3,0)-2CC	0.64	0.71	0.74	0.77	—	16	GJR(3,0)-2SEC	0.72	0.78	0.81	0.83	ii, v	23
GJR(3,0)-3PC	0.76	0.80	0.84	0.86	i, iii, v	22	GJR(3,0)-3CC	0.41	0.50	0.54	0.58	-	12	GJR(3,0)-3SEC	0.72	0.77	0.80	0.82	ii, v	23
GJR(5,0)-1PC	0.71	0.74	0.77	0.79	ii	26	GJR(5,0)-1CC	0.74	0.77	0.80	0.82	ii	23	GJR(5,0)-1SEC	0.63	0.67	0.70	0.72	—	25
GJR(5,0)-2PC	0.72	0.76	0.79	0.81	ii, iv, v	25	GJR(5,0)-2CC	0.60	0.66	0.69	0.72	—	18	GJR(5,0)-2SEC	0.67	0.73	0.75	0.78	ii, v	26
GJR(5,0)-3PC	0.71	0.75	0.79	0.81	i, iii, v	25	GJR(5,0)-3CC	0.41	0.49	0.54	0.58	—	10	GJR(5,0)-3SEC	0.67	0.72	0.75	0.77	ii, v	25

Note: The table shows the cumulative volatility explained by the first four principal components of each PVCF model. The first element of the model name indicates the ARCH process employed for the idiosyncrasies, followed by the number of variance factors obtained by Principal Components (PC), Canonical Correlations (CC) and Economic Sectors (SEC). Covariance Tests represent the combination of covariance tests that each model passes up to the second lag at 5%. The combinations are: (i) $CLC^{1,2}$, (ii) CLC^{2}, (iii) $ACLC^{2,3}$, (iv) $ACLC^{3}$, (v) $AALC^{2,3}$, (vi) $AALC^{3}$, where the superscripts indicate the lag. '—' signifies that none of such tests is passed. LB^2 is the Ljung–Box test for the squared standardized residuals and gives the number of assets (out of 30) for which one cannot reject the null of absence of serial correlation until the 15th lag at 5%.

Table 7
Comparison of the PVCF models with market factors

Model	1pc	2pc	3pc	4pc	Covariance tests	LB^2	Model	1pc	2pc	3pc	4pc	Covariance tests	LB^2	Model	1pc	2pc	3pc	4pc	Covariance tests	LB^2
GARCH(1,0)-NQ	0.85	0.88	0.90	0.91	—	7	EGARCH(1,0)-NQ	0.86	0.88	0.90	0.92	—	8	GJR(1,0)-NQ	0.82	0.85	0.88	0.90	—	7
GARCH(1,0)-VX	0.79	0.81	0.84	0.86	—	9	EGARCH(1,0)-VX	0.83	0.85	0.88	0.89	—	8	GJR(1,0)-VX	0.76	0.79	0.82	0.84	—	8
GARCH(1,0)-DJ	0.84	0.87	0.89	0.90	—	12	EGARCH(1,0)-DJ	0.84	0.88	0.90	0.91	—	10	GJR(1,0)-DJ	0.81	0.84	0.86	0.88	—	13
GARCH(1,0)-SP	0.83	0.86	0.88	0.90	—	10	EGARCH(1,0)-SP	0.84	0.87	0.89	0.91	—	7	GJR(1,0)-SP	0.80	0.83	0.85	0.88	—	10
GARCH(1,0)-NQVX	0.80	0.85	0.88	0.90	—	12	EGARCH(1,0)-NQVX	0.83	0.87	0.89	0.91	—	8	GJR(1,0)-NQVX	0.78	0.83	0.86	0.88	—	11
GARCH(1,0)-NQDJ	0.82	0.88	0.91	0.92	—	13	EGARCH(1,0)-NQDJ	0.83	0.88	0.90	0.92	—	10	GJR(1,0)-NQDJ	0.79	0.85	0.87	0.90	—	13
GARCH(1,0)-DJVX	0.81	0.83	0.86	0.87	—	11	EGARCH(1,0)-DJVX	0.84	0.86	0.88	0.90	—	8	GJR(1,0)-DJVX	0.77	0.80	0.83	0.85	—	10
GARCH(1,0)-DJSP	0.82	0.85	0.87	0.89	—	12	EGARCH(1,0)-DJSP	0.83	0.86	0.88	0.90	—	9	GJR(1,0)-DJSP	0.78	0.82	0.85	0.87	—	13
GARCH(1,0)-SPNQ	0.81	0.87	0.90	0.92	—	12	EGARCH(1,0)-SPNQ	0.83	0.88	0.90	0.92	—	9	GJR(1,0)-SPNQ	0.78	0.84	0.87	0.89	—	13
GARCH(1,0)-SPVX	0.80	0.83	0.86	0.88	—	10	EGARCH(1,0)-SPVX	0.83	0.86	0.88	0.90	—	7	GJR(1,0)-SPVX	0.77	0.81	0.83	0.85	—	10
GARCH(1,0)-VXSP	0.78	0.84	0.87	0.89	—	11	EGARCH(1,0)-VXSP	0.81	0.86	0.89	0.91	—	8	GJR(1,0)-VXSP	0.76	0.82	0.85	0.87	—	11
GARCH(1,0)-NQVXDJ	0.79	0.85	0.88	0.90	—	11	EGARCH(1,0)-NQVXDJ	0.82	0.87	0.89	0.91	—	9	GJR(1,0)-NQVXDJ	0.76	0.83	0.85	0.88	—	12
GARCH(1,0)-DJSPNQ	0.81	0.87	0.89	0.91	—	14	EGARCH(1,0)-DJSPNQ	0.82	0.87	0.90	0.91	—	11	GJR(1,0)-DJSPNQ	0.77	0.83	0.86	0.88	—	14
GARCH(1,0)-DJVXSP	0.77	0.82	0.84	0.86	—	10	EGARCH(1,0)-DJVXSP	0.81	0.85	0.88	0.89	—	8	GJR(1,0)-DJVXSP	0.75	0.80	0.82	0.84	—	10
GARCH(3,0)-NQ	0.80	0.83	0.85	0.87	—	21	EGARCH(3,0)-NQ	0.77	0.81	0.83	0.85	—	20	GJR(3,0)-NQ	0.77	0.80	0.83	0.84	—	21
GARCH(3,0)-VX	0.72	0.75	0.78	0.80	—	18	EGARCH(3,0)-VX	0.74	0.77	0.79	0.81	—	13	GJR(3,0)-VX	0.70	0.73	0.76	0.78	—	19
GARCH(3,0)-DJ	0.78	0.80	0.82	0.84	—	20	EGARCH(3,0)-DJ	0.75	0.78	0.81	0.83	—	20	GJR(3,0)-DJ	0.74	0.77	0.79	0.81	—	19
GARCH(3,0)-SP	0.78	0.80	0.83	0.84	—	23	EGARCH(3,0)-SP	0.75	0.78	0.81	0.83	—	20	GJR(3,0)-SP	0.75	0.78	0.80	0.82	—	22
GARCH(3,0)-NQVX	0.76	0.82	0.84	0.86	—	23	EGARCH(3,0)-NQVX	0.76	0.81	0.83	0.86	—	23	GJR(3,0)-NQVX	0.74	0.79	0.82	0.85	—	23
GARCH(3,0)-NQDJ	0.79	0.84	0.87	0.89	ii, vi	23	EGARCH(3,0)-NQDJ	0.77	0.82	0.84	0.86	ii, vi	25	GJR(3,0)-NQDJ	0.76	0.82	0.84	0.86	ii, vi	24
GARCH(3,0)-DJVX	0.75	0.78	0.80	0.82	—	21	EGARCH(3,0)-DJVX	0.75	0.78	0.81	0.83	—	18	GJR(3,0)-DJVX	0.72	0.75	0.78	0.80	—	21
GARCH(3,0)-DJSP	0.76	0.79	0.82	0.84	—	23	EGARCH(3,0)-DJSP	0.74	0.77	0.80	0.82	—	20	GJR(3,0)-DJSP	0.73	0.76	0.79	0.81	—	23
GARCH(3,0)-SPNQ	0.78	0.83	0.86	0.88	ii, vi	23	EGARCH(3,0)-SPNQ	0.77	0.81	0.84	0.86	ii, vi	24	GJR(3,0)-SPNQ	0.75	0.81	0.84	0.86	ii, vi	23

GARCH(3,0)-SPVX	0.75	0.78	0.81	0.83	—	22	EGARCH(3,0)-SPVX	0.74	0.78	0.81	0.83	—	18	GJR(3,0)-SPVX	0.72	0.76	0.78	0.81	—	21
GARCH(3,0)-VXSP	0.75	0.81	0.84	0.86	—	24	EGARCH(3,0)-VXSP	0.75	0.79	0.83	0.85	—	24	GJR(3,0)-VXSP	0.73	0.79	0.82	0.84	—	24
GARCH(3,0)-NQVXDJ	0.75	0.81	0.84	0.86	—	24	EGARCH(3,0)-NQVXDJ	0.75	0.80	0.83	0.85	—	22	GJR(3,0)-NQVXDJ	0.73	0.79	0.82	0.84	—	24
GARCH(3,0)-DJSPNQ	0.77	0.83	0.86	0.88	ii, vi	24	EGARCH(3,0)-DJSPNQ	0.76	0.80	0.83	0.85	ii, vi	25	GJR(3,0)-DJSPNQ	0.75	0.80	0.83	0.85	ii, vi	24
GARCH(3,0)-DJVXSP	0.72	0.77	0.79	0.82	—	22	EGARCH(3,0)-DJVXSP	0.73	0.77	0.80	0.82	—	18	GJR(3,0)-DJVXSP	0.70	0.75	0.78	0.80	—	23
GARCH(5,0)-NQ	0.76	0.80	0.82	0.84	—	25	EGARCH(5,0)-NQ	0.74	0.77	0.80	0.82	ii	22	GJR(5,0)-NQ	0.72	0.75	0.78	0.80	—	23
GARCH(5,0)-VX	0.67	0.71	0.73	0.76	—	24	EGARCH(5,0)-VX	0.70	0.73	0.76	0.78	—	21	GJR(5,0)-VX	0.66	0.69	0.72	0.75	—	24
GARCH(5,0)-DJ	0.73	0.76	0.78	0.80	—	25	EGARCH(5,0)-DJ	0.71	0.74	0.77	0.79	—	24	GJR(5,0)-DJ	0.71	0.74	0.76	0.78	—	25
GARCH(5,0)-SP	0.74	0.77	0.79	0.81	—	26	EGARCH(5,0)-SP	0.71	0.74	0.77	0.79	—	24	GJR(5,0)-SP	0.71	0.74	0.76	0.78	—	26
GARCH(5,0)-NQVX	0.71	0.76	0.79	0.81	—	23	EGARCH(5,0)-NQVX	0.74	0.78	0.81	0.83	—	23	GJR(5,0)-NQVX	0.71	0.75	0.78	0.80	—	24
GARCH(5,0)-NQDJ	0.75	0.81	0.83	0.85	ii	26	EGARCH(5,0)-NQDJ	0.74	0.78	0.81	0.83	ii, vi	25	GJR(5,0)-NQDJ	0.73	0.78	0.81	0.83	ii, vi	26
GARCH(5,0)-DJVX	0.70	0.74	0.76	0.78	—	24	EGARCH(5,0)-DJVX	0.72	0.75	0.77	0.79	—	22	GJR(5,0)-DJVX	0.68	0.72	0.74	0.76	—	24
GARCH(5,0)-DJSP	0.71	0.75	0.77	0.79	—	26	EGARCH(5,0)-DJSP	0.70	0.74	0.76	0.78	—	24	GJR(5,0)-DJSP	0.69	0.72	0.75	0.77	—	25
GARCH(5,0)-SPNQ	0.76	0.81	0.84	0.86	ii	27	EGARCH(5,0)-SPNQ	0.73	0.78	0.80	0.83	ii, vi	25	GJR(5,0)-SPNQ	0.73	0.78	0.81	0.83	ii, vi	27
GARCH(5,0)-SPVX	0.70	0.74	0.76	0.79	—	25	EGARCH(5,0)-SPVX	0.71	0.75	0.78	0.80	—	23	GJR(5,0)-SPVX	0.69	0.73	0.75	0.78	—	25
GARCH(5,0)-VXSP	0.71	0.77	0.80	0.82	—	25	EGARCH(5,0)-VXSP	0.72	0.77	0.80	0.82	—	25	GJR(5,0)-VXSP	0.69	0.74	0.77	0.80	—	25
GARCH(5,0)-NQVXDJ	0.73	0.79	0.82	0.84	—	25	EGARCH(5,0)-NQVXDJ	0.72	0.77	0.80	0.82	—	24	GJR(5,0)-NQVXDJ	0.71	0.77	0.80	0.82	—	25
GARCH(5,0)-DJSPNQ	0.72	0.78	0.81	0.83	ii	26	EGARCH(5,0)-DJSPNQ	0.72	0.77	0.80	0.82	ii, vi	24	GJR(5,0)-DJSPNQ	0.70	0.76	0.79	0.81	ii, vi	27
GARCH(5,0)-DJVXSP	0.68	0.72	0.75	0.77	—	24	EGARCH(5,0)-DJVXSP	0.70	0.74	0.77	0.79	—	24	GJR(5,0)-DJVXSP	0.66	0.70	0.73	0.75	—	25

Note: The table shows the cumulative volatility explained by the first four principal components of each PVCF-MKT model. The first element of the model name indicates the ARCH process employed for the idiosyncrasies, followed by the variance factors which are the volatilities of the Dow Jones (DJ), NASDAQ (NQ), S&P500 (SP) indices or the VIX (VX). Covariance Tests represent the combination of covariance tests that each model passes up to the second lag at 5%. The combinations are: (i) $CLC^{1,2}$, (ii) CLC^{2}, (iii) $ACLC^{2,3}$, (iv) $ACLC^{3}$, (v) $AALC^{2,3}$, (vi) $AALC^{3}$, where the superscripts indicate the lag. '—' signifies that none of such tests is passed. LB^2 is the Ljung–Box test for the squared standardized residuals and gives the number of assets (out of 30) for which one cannot reject the null of absence of serial correlation until the 15th lag at 5%.

and those with variance factors from the market indices and from the economic sectors (PVCF-MKT and PVCF-SEC). Five robust covariance tests (LC, ALC, CLC^k, $ACLC^k$ and $AALC^k$ tests) up to the fifth lag are evaluated for each model using HAC robust standard errors for the empirical moments as in (28), where an automatic truncation lag is adopted. However, we report only those robust tests passed by each model. The CLC tests at higher lags are constructed using the kth dimensional vector of empirical moments given by the sum of the sample covariances from lag 1 to k. One possible problem with the LC and ALC tests is the high number of degrees of freedom (900 and 435, respectively, with $N = 30$) for which the asymptotic theory may not work properly. Consequently, in commenting the results, we put more emphasis on the other covariance tests.

The variance explained by the first principal component of the conditional volatilities is quite high (always greater than 40%), but the next principal components can hardly explain the 5%, suggesting that there is no need for additional variance factors. The Ljung–Box test for the squared standardized residuals (LB^2) indicates that the PVCF-PC models capture most of the individual ARCH effects in each asset. When the number of lags taken for the ARCH process of the idiosyncratic volatilities is high, almost every return can pass the LB^2 test.

Combining the different criteria shown in Tables 6 and 7 and looking at those models that pass most of the covariance tests, we select the best model within each PVCF model. The EGARCH(5,0)-3PC is the best PVCF-PC model, while the GJR(5,0)-1CC and the GARCH(5,0)-2SEC outperform all the others in the PVCF-CC and PVCF-SEC classes, respectively. The GJR(5,0)-SPNQ turns out to be the best model among those that employ market variance factors. All these models are characterized by the highest number of acceptances for the LB^2 test, and the highest proportion of explained variance. In addition, the EGARCH(5,0)-3PC model seems to a certain extent superior among the best models in terms of number of covariance tests passed. However, no model can clearly beat the others. In terms of number of variance features, the PVCF-CC model is the only one for which one variance factor is sufficient to characterize the comovements in the volatilities of the Dow. The additive model both with symmetric or asymmetric GARCH for the idiosyncratic volatilities seems to perform better than the multiplicative model. Actually, the PVCF models with GARCH and GJR processes for the transitory volatility component pass the covariance tests more often than those with EGARCH. Tables 8–11 give the estimated parameters for the best model in each class of PVCF model. Only the estimates for the long-run volatility components are presented.

Table 8 exhibits the parameter estimates for the GJR(5,0)-SPNQ model with two variance factors given by the GARCH volatilities of the S&P500 and NASDAQ indices. For almost all assets in the Dow, the variance factors are highly significant. Moreover, the model rejects the null of absence of serial correlation in the squared standardized residuals up to the 15th lag only for few assets (Johnson & Johnson, SBC Communications and Wal-Mart Stores). The GJR(5,0)-SPNQ model does not pass the LC, ALC and CLC tests at the first lag and ACLC at the second lag at any reasonable significance level. Instead, it passes the CLC tests at the second through

Table 8
Estimation results for the best PVCF-MKT model: the GJR(5,0)-SPNQ

	SP500 Vol. Fact.	NASDAQ Vol. Fact.	LB^2		SP500 Vol. Fact.	NASDAQ Vol. Fact.	LB^2		SP500 Vol. Fact.	NASDAQ Vol. Fact.	LB^2
AA	−0.1656	0.7615	OK	HD	0.0930	0.6993	OK	MMM	0.7398	0.1102	OK
	(0.3310)	(0.1280)			(0.3435)	(0.1194)			(0.2935)	(0.0657)	
AXP	1.7893	0.2527	OK	HON	1.0339	0.4819	OK	MO	2.3727	0.0095	OK
	(0.5287)	(0.1056)			(0.5689)	(0.1341)			(0.5798)	(0.1210)	
BA	2.1722	0.0767	OK	HPQ	0.7139	1.1210	OK	MRK	1.0406	−0.0174	OK
	(0.6359)	(0.1248)			(0.8526)	(0.2405)			(0.2842)	(0.0676)	
CAT	1.7831	0.0244	OK	IBM	0.7057	0.4451	OK	MSFT	−0.4250	0.7604	OK
	(0.5204)	(0.1006)			(0.6139)	(0.1746)			(0.3966)	(0.1854)	
CITI	1.6516	−0.0683	OK	INTC	−0.3426	1.2228	OK	PG	0.9061	0.1522	OK
	(0.5129)	(0.1227)			(0.5456)	(0.2256)			(0.3102)	(0.0958)	
DD	1.8863	0.1034	OK	IP	1.5059	0.2309	OK	SBC	0.8445	0.2906	9
	(0.4020)	(0.1066)			(0.4258)	(0.1095)			(0.3038)	(0.1063)	
DIS	0.8323	0.4647	OK	JNJ	0.4354	−0.0305	6	T	1.0560	0.5777	OK
	(0.4876)	(0.1639)			(0.2344)	(0.0435)			(0.5362)	(0.1368)	
EK	0.8932	0.0275	OK	JPM	1.0346	0.5103	OK	UTX	0.9500	0.1792	OK
	(0.4669)	(0.0950)			(0.5148)	(0.1536)			(0.2490)	(0.0617)	
GE	1.2857	0.1429	OK	KO	0.9148	0.1317	OK	WMT	0.5756	0.4221	10
	(0.2964)	(0.0680)			(0.2778)	(0.0750)			(0.3195)	(0.1088)	
GM	0.1886	0.3131	OK	MCD	0.7008	0.1933	OK	XOM	1.2691	−0.0435	OK
	(0.2678)	(0.0811)			(0.2832)	(0.0699)			(0.2027)	(0.0482)	
Covariance tests											
LC^1	*p*-value	CLC^1	*p*-value	CLC^2	*p*-value	CLC^3	*p*-value	CLC^4	*p*-value	CLC^5	*p*-value
6013.85	0	10.68	0	3.83	0.05	2.26	0.13	2.36	0.12	4.98	0.03
		ALC^1	*p*-value	$ACLC^2$	*p*-value	$ACLC^3$	*p*-value	$AALC^2$	*p*-value	$AALC^3$	*p*-value
		1071.67	0	11.21	0	12.02	0.01	7.71	0.02	7.87	0.05

Note: The estimated PVCF model is a GJR(5,0) for the short-run volatility component (whose estimates are not reported) and two market volatility factors, given by the estimated volatility of the S&P500, and the NASDAQ. Robust standard errors (as in Bollerslev and Wooldridge, 1992) are in parentheses. LB^2 represents the Ljung–Box test for the absence of serial correlation in the standardized squared residuals up to 15 lags. For each asset, the first lag at which one rejects the null at 5% is reported. Alternatively, an OK indicates that such lag is not less than 15. The superscripts in the covariance tests indicate the number of lags.

Table 9
Estimation results for the best PVCF-PC model: the EGARCH(5,0)-3PC Model

	First PC Vol. Fact.	Second PC Vol. Fact.	Third PC Vol. Fact.	LB^2		First PC Vol. Fact.	Second PC Vol. Fact.	Third PC Vol. Fact.	LB^2		First PC Vol. Fact.	Second PC Vol. Fact.	Third PC Vol. Fact.	LB^2
AA	0.2180	0.1929	0.5636	OK	HD	0.2357	0.3438	0.3587	OK	MMM	0.5240	0.0482	0.3642	OK
	(0.0872)	(0.1022)	(0.1052)			(0.0991)	(0.1032)	(0.0917)			(0.0903)	(0.1004)	(0.1052)	
AXP	0.4552	0.3304	0.2053	OK	HON	0.5080	0.1412	0.4245	OK	MO	0.5338	0.4370	−0.0567	OK
	(0.1095)	(0.0875)	(0.0999)			(0.1424)	(0.1165)	(0.1086)			(0.1097)	(0.1221)	(0.1979)	
BA	0.6942	0.1663	0.0988	OK	HPQ	0.2753	0.5444	0.1151	OK	MRK	0.1570	−0.0249	0.4425	OK
	(0.1110)	(0.1108)	(0.1320)			(0.0858)	(0.0822)	(0.0941)			(0.0926)	(0.1022)	(0.1112)	
CAT	0.3840	−0.0163	0.3278	OK	IBM	0.1786	0.3377	0.3335	OK	MSFT	0.1263	0.4933	0.1454	OK
	(0.0939)	(0.0995)	(0.0911)			(0.1114)	(0.1296)	(0.1449)			(0.0804)	(0.1115)	(0.0968)	
CITI	0.4071	−0.0705	0.2064	OK	INTC	0.1722	0.3146	0.3491	OK	PG	0.4598	0.1604	0.2648	OK
	(0.0998)	(0.0976)	(0.1080)			(0.0871)	(0.1137)	(0.0940)			(0.1067)	(0.1138)	(0.0983)	
DD	0.5460	−0.0191	0.4586	OK	IP	0.4591	0.2069	0.3828	OK	SBC	0.5251	0.3473	0.1881	6
	(0.0878)	(0.0884)	(0.1148)			(0.0793)	(0.0960)	(0.1181)			(0.0812)	(0.0988)	(0.1011)	
DIS	0.4595	0.2493	0.2816	OK	JNJ	0.0737	−0.0459	0.3686	5	T	0.6091	0.2280	0.4422	OK
	(0.1018)	(0.1334)	(0.1021)			(0.0922)	(0.0913)	(0.0970)			(0.1315)	(0.1157)	(0.1526)	
EK	0.3449	−0.0255	0.2280	OK	JPM	0.3956	0.1824	0.3990	OK	UTX	0.4418	0.0628	0.5316	OK
	(0.1471)	(0.1294)	(0.1965)			(0.0940)	(0.1120)	(0.1002)			(0.0835)	(0.0868)	(0.0863)	
GE	0.7621	0.2861	0.0232	OK	KO	0.3427	0.2273	0.2847	OK	WMT	0.2353	0.3094	0.3959	10
	(0.0739)	(0.0839)	(0.0897)			(0.0876)	(0.1136)	(0.1200)			(0.0772)	(0.0939)	(0.0966)	
GM	0.1544	0.0280	0.4101	OK	MCD	0.2409	0.1027	0.5447	5	XOM	0.5169	0.1040	0.2749	OK
	(0.0853)	(0.0967)	(0.0833)			(0.1259)	(0.1129)	(0.0943)			(0.0959)	(0.1012)	(0.1014)	
Covariance tests														
		LC^1	*p*-value	CLC^1	*p*-value	CLC^2	*p*-value	CLC^3	*p*-value	CLC^4	*p*-value	CLC^5	*p*-value	
		5722.17	0	1.97	0.16	0.17	0.68	0.01	0.93	0.01	0.9	1.52	0.22	
				ALC^1	*p*-value	$ACLC^2$	*p*-value	$ACLC^3$	*p*-value	$AALC^2$	*p*-value	$AALC^3$	*p*-value	
				1073	0	2.23	0.33	2.26	0.52	1.66	0.43	2.12	0.55	

Note: The estimated PVCF model is an EGARCH(5,0) for the short-run volatility component (not reported) and three variance factors, represented by the estimated conditional volatilities of the three largest principal components of the returns. The variance factors are in logarithms because EGARCH models the log of the conditional variance. Robust standard errors (as in Bollerslev and Wooldridge, 1992) are in parentheses. LB^2 represents the Ljung–Box test for the absence of serial correlation in the standardized squared residuals up to 15 lags. For each asset, the first lag at which one rejects the null at 5% is reported. Alternatively, an OK indicates that such lag is not less than 15. The superscripts in the covariance tests indicate the number of lags.

Table 10
Estimation results for the best PVCF-CC model: the GJR(5,0)-1CC Model

	First CC Vol. Fact.	LB^2		First CC Vol. Fact.	LB^2		First CC Vol. Fact.	LB^2
AA	1.0861	12	HD	1.6068	OK	MMM	0.8250	OK
	(0.1629)			(0.1460)			(0.0896)	
AXP	1.4066	7	HON	1.2149	OK	MO	0.9573	OK
	(0.1364)			(0.2031)			(0.1689)	
BA	1.0395	OK	HPQ	0.2968	OK	MRK	0.9062	OK
	(0.2247)			(0.2601)			(0.1078)	
CAT	0.4575	OK	IBM	0.8823	OK	MSFT	1.0509	OK
	(0.1100)			(0.2201)			(0.1965)	
CITI	1.0450	OK	INTC	1.3723	OK	PG	0.8665	OK
	(0.1491)			(0.2008)			(0.0866)	
DD	1.0873	OK	IP	1.2376	OK	SBC	0.7167	4
	(0.1237)			(0.1364)			(0.0944)	
DIS	1.0018	OK	JNJ	0.4236	10	T	1.1863	OK
	(0.1552)			(0.0796)			(0.1318)	
EK	0.3624	OK	JPM	1.5365	OK	UTX	0.7270	OK
	(0.0939)			(0.1653)			(0.0892)	
GE	0.8178	7	KO	0.6907	9	WMT	1.3186	10
	(0.0903)			(0.0727)			(0.1259)	
GM	0.7941	OK	MCD	0.8479	OK	XOM	0.4098	OK
	(0.1361)			(0.1064)			(0.0575)	
Covariance tests								
	LC^1	*p*-value	CLC^1	*p*-value	CLC^2	*p*-value	CLC^3	*p*-value
	6467.73	0	18.05	0	2.49	0.11	8.3	0
	CLC^4	*p*-value	CLC^5	*p*-value	ALC^1	*p*-value	$ACLC^2$	*p*-value
	2.04	0.15	0.14	0.71	1125	0	45.16	0
			$ACLC^3$	*p*-value	$AALC^2$	*p*-value	$AALC^3$	*p*-value
			46.08	0	12.9	0	24.15	0

Note: The estimated PVCF model is a GJR(5,0) for the short-run volatility component (not reported) and one variance factor given by the exponential of the canonical variate associated with the largest canonical correlation between the squared returns in logarithm and their most recent past. Robust standard errors (as in Bollerslev and Wooldridge, 1992) are in parentheses. LB^2 represents the Ljung–Box test for the absence of serial correlation in the standardized squared residuals up to 15 lags. For each asset, the first lag at which one rejects the null at 5% is reported. Alternatively, an OK indicates that such lag is not less than 15. The superscripts in the covariance tests indicate the number of lags.

the fourth lag at 5%, the CLC at the fifth, the ACLC at the third and the AALC at the second lag at 1%.

Table 9 shows the results for the EGARCH(5,0)-3PC model with three variance factors given by the Component ARCH volatilities of the first three principal components of the returns. For most returns, the volatility factors are significant and there is some degree of serial correlation left in the squared standardized residuals, because we fail to reject the null up to the 15th lag for all the assets but Johnson & Johnson, McDonalds, SBC Communications and Wal-Mart Stores. The EGARCH(5,0)-3PC model cannot pass the LC and the ALC tests at any level of

Table 11
Estimation results for the best PVCF-SEC model: the GARCH(5,0)-2SEC Model

	First SEC Vol. Fact.	Second SEC Vol. Fact.	LB^2		First SEC Vol. Fact.	Second SEC Vol. Fact.	LB^2		First SEC Vol. Fact.	Second SEC Vol. Fact.	LB^2
AA	0.0862	1.2417	9	HD	0.0932	1.7058	OK	MMM	−0.0194	0.0837	OK
	(0.0578)	(0.3037)			(0.0596)	(0.3872)			(0.0012)	(0.0408)	
AXP	0.1849	1.5697	OK	HON	−0.0102	2.1070	OK	MO	0.5651	−0.2178	OK
	(0.0647)	(0.5046)			(0.0536)	(0.5206)			(0.0920)	(0.1457)	
BA	0.3027	0.5148	OK	HPQ	−0.0009	0.3680	OK	MRK	0.2180	0.0969	OK
	(0.0744)	(0.2990)			(0.0923)	(0.6145)			(0.0562)	(0.2265)	
CAT	0.1607	0.6947	OK	IBM	0.2803	0.9308	OK	MSFT	0.0501	1.4576	OK
	(0.0610)	(0.2752)			(0.0723)	(0.3274)			(0.0719)	(0.3530)	
CITI	0.1994	0.5137	OK	INTC	0.0812	2.2936	OK	PG	0.2425	0.3231	OK
	(0.0802)	(0.3283)			(0.0908)	(0.4889)			(0.0485)	(0.1909)	
DD	0.2571	0.7916	OK	IP	0.2770	0.6650	OK	SBC	0.1922	0.8158	9
	(0.0502)	(0.2488)			(0.0544)	(0.2629)			(0.0386)	(0.1663)	
DIS	0.0822	1.8441	OK	JNJ	0.1111	−0.0708	9	T	0.2237	0.8872	OK
	(0.0607)	(0.3744)			(0.0447)	(0.1015)			(0.0542)	(0.2843)	
EK	0.0895	0.4618	OK	JPM	0.0666	2.0650	OK	UTX	0.1066	0.8663	OK
	(0.0823)	(0.4062)			(0.0573)	(0.3629)			(0.0388)	(0.2165)	
GE	0.1427	1.1514	OK	KO	0.2019	0.3865	OK	WMT	0.2915	0.6345	10
	(0.0360)	(0.2952)			(0.0402)	(0.1504)			(0.0507)	(0.2277)	
GM	0.0099	1.0764	OK	MCD	0.1117	0.6480	OK	XOM	0.1474	0.2143	OK
	(0.0467)	(0.2999)			(0.0379)	(0.1685)			(0.0348)	(0.1220)	
Covariance tests											
LC^1	*p*-value	CLC^1	*p*-value	CLC^2	*p*-value	CLC^3	*p*-value	CLC^4	*p*-value	CLC^5	*p*-value
5766.74	0	9.02	0	2.95	0.09	2.14	0.14	2.14	0.14	4.49	0.03
		ALC^1	*p*-value	$ACLC^2$	*p*-value	$ACLC^3$	*p*-value	$AALC^2$	*p*-value	$AALC^3$	*p*-value
		1168.72	0	9.28	0.01	9.59	0.02	6.14	0.05	6.15	0.1

Note: The estimated PVCF model is a GARCH(5,0) for the short-run volatility component (not reported) and two variance factors, which are given by the conditional volatilities of the two largest principal components of the average returns of the Dow's sectors. Robust standard errors (as in Bollerslev and Wooldridge, 1992) are in parentheses. LB^2 represents the Ljung–Box test for the absence of serial correlation in the standardized squared residuals up to 15 lags. For each asset, the first lag at which one rejects the null at 5% is reported. Alternatively, an OK indicates that such lag is not less than 15. The superscripts in the covariance tests indicate the number of lags.

significance. However, it passes the CLC test from the first up to the fifth lag and all the ACLC and AALC tests at 5%. Thus, the model shows evidence of adequacy in capturing the common volatilities in the thirty stocks of the Dow Jones.

Table 10 illustrates the estimates for the GJR(5,0)-1CC model with just one variance feature given by the exponential of the canonical variate associated with the largest canonical correlation between the log-transformed squared returns and their most recent past. The volatility factor is highly significant for all stock returns. Seven stocks (Alcoa, American Express, General Electric, Johnson & Johnson, Coca Cola, SBC Communications, and Wal-Mart Stores) show evidence of residual serial correlation in the squared standardized returns. The GJR(5,0)-1CC model cannot pass either the LC, the ALC and the CLC tests at first and third lag or all the additive tests. Nevertheless, it passes the CLC tests at the second, fourth and fifth lag at the usual 5% significance level.

Table 11 exhibits the estimates for the GARCH(5,0)-2SEC model with two variance factors given by the GARCH volatilities of the first two principal components of the returns of the Dow Jones' sectors. Only few stocks present estimates for the factor volatilities that are not significant. Some serial correlation in the squared standardized residuals can be found only in Alcoa, Johnson & Johnson, SBC Communications and Wal-Mart Stores. The GARCH(5,0)-2SEC model does not pass the LC, the ALC, and the CLC tests at the first lag at any reasonable significance level. However, it passes the CLC tests at the second through the fifth lag and the AALC at the second lag at 5%. It also passes the ACLC at all lags at 5%.

Fig. 2 depicts the factors and the corresponding volatility factors obtained from Canonical Correlations and Principal Components. The top panel of Fig. 3 illustrates the variance factors from the stock market indices (Dow Jones, NASDAQ, S&P500 and VIX), while the bottom panels show the factors and the variance factors from the economic sectors. We can see how dissimilar the variance factors are, but for the conditional volatility of the first principal component which mimics the Dow Jones' volatility. All the other variance factors show very different volatility patterns.

All these results confirm that few variance factors (between one and three) can well characterize the comovements in the volatility process of the Dow Jones. Therefore, together with a market volatility factor, we have evidence of few more variance features that can be related to particular economic sectors. Moreover, since the explained variability of the principal components next to the first is really tiny, additional factors seem superfluous to describe further common volatilities.

The PVCF-CC model turns out to be the most parsimonious because only one variance factor is sufficient to capture the comovements in all the volatilities of the Dow. Although the PVCF-PC model requires the highest number of factors, it passes most of the covariance tests aimed to detect the time dependence left on the residuals. In general, no model seems to clearly outperform the competitors, but several low-factor models give an adequate representation of the common volatilities of the Dow.

6. Conclusions

In this paper, we present a new model to characterize the comovements in the volatilities of a portfolio of stock returns. We call it the LRPVCF model, because we apply the Engle and Kozicki (1993) CF framework to the analysis of the long-run forecasts of the second moments of a portfolio of assets. The PVCF model is a linear factor model for the conditional variances, which is able to isolate those common volatilities that drive all the long-run comovements in the second moments. It decomposes the volatility process into a short-run (or idiosyncratic) component, which is modeled with a low-order ARCH, and a long-run volatility component, which is modeled through a factor structure. The ARCH process for the idiosyncrasies is used to model possible time-varying idiosyncratic volatilities, which could be the main cause of misleading results from the common ARCH tests.

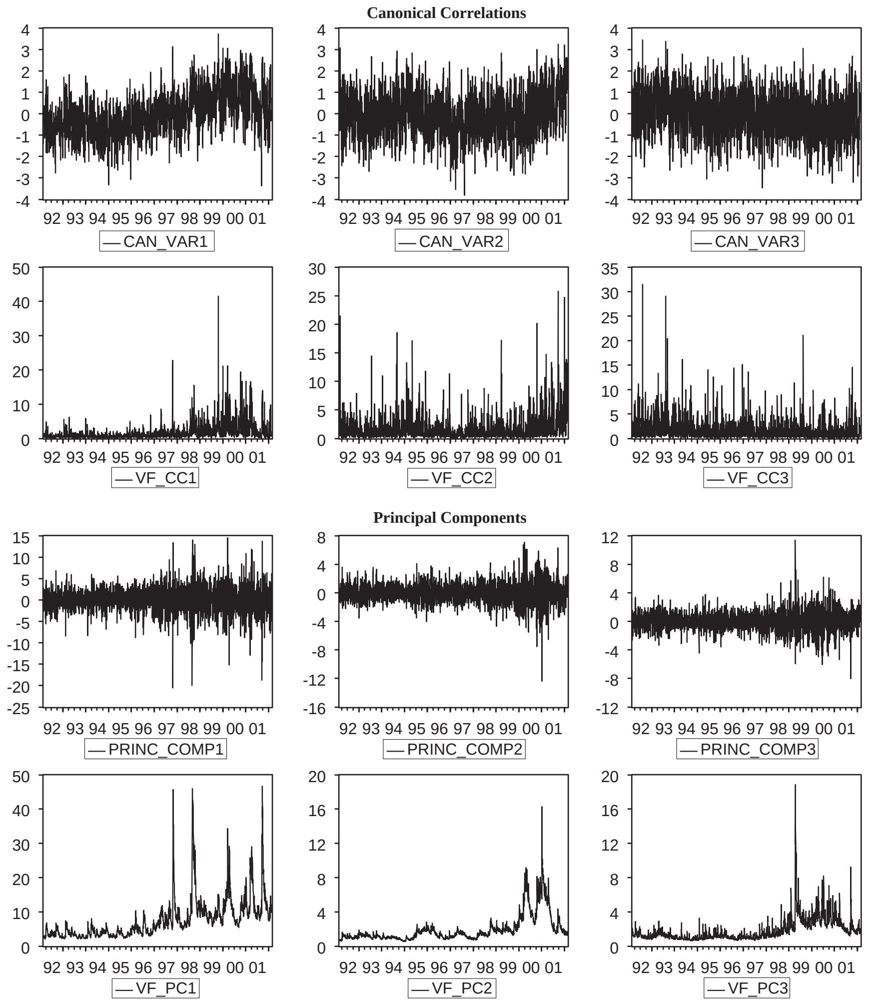

Fig. 2. Factors and factor volatilities from statistical approaches. *Notes*: The top panel shows the canonical variates associated with the three largest canonical correlations between the log-squared returns and their most recent past and the corresponding variance factors computed as the exponentials of each variate. The bottom panel depicts the first three principal components of the Dow Jones returns and their Component-ARCH volatilities.

To identify the number of PVCF, we use the Bartlett test for the significance of the smallest canonical correlations, typically used in reduced-rank regression. In the empirical application, this method suggests that there are at most three variance factors in the thirty volatilities of the Dow Jones Industrial Index.

We thus estimate different PVCF models, modeling the short-run component with an ARCH process with at most five lags and the long-run component with one–three

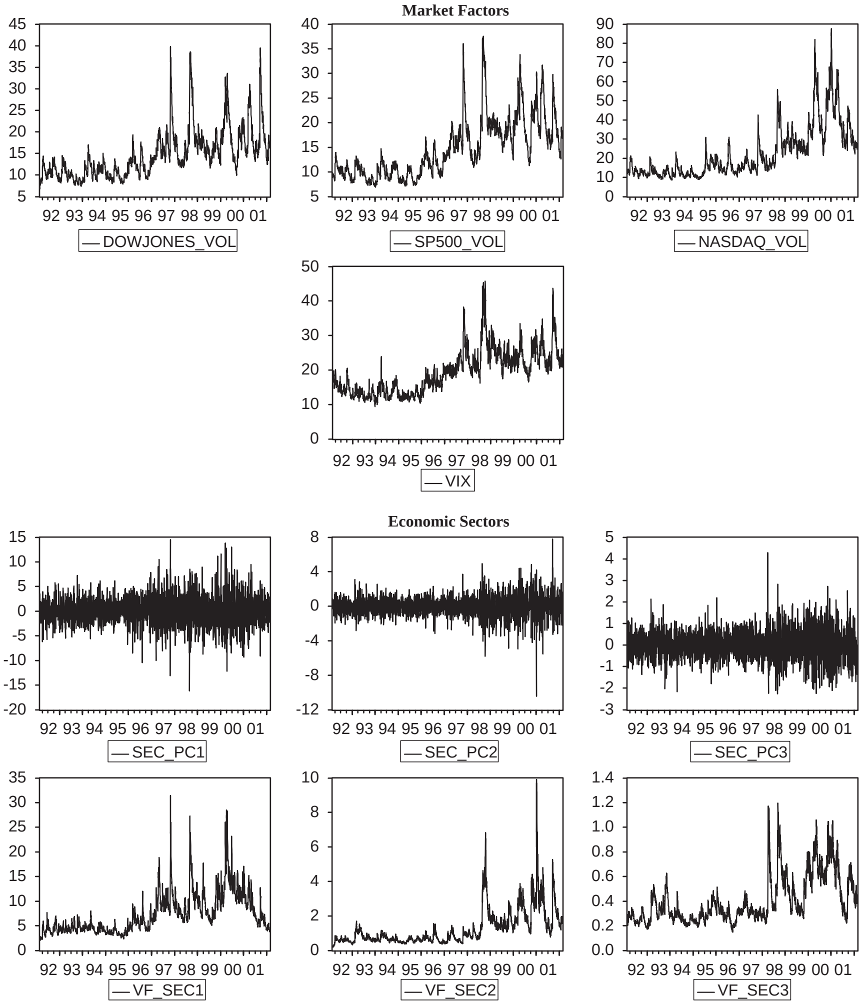

Fig. 3. Factors and factor volatilities from observable approaches. *Notes*: The top panel illustrates the market variance factors computed as the annualized GARCH volatilities of Dow Jones, S&P500, NASDAQ and the new VIX. The bottom panel shows the first three principal components of the returns of the Dow Jones' sectors, together with their GARCH volatilities.

factors. The pure variance common factors are specified using both statistical methods, such as principal component and canonical correlation analysis, and observable approaches, where the volatilities of few major US stock market indices (Dow Jones, NASDAQ, S&P500) together with the VIX are utilized. In addition, the volatilities of the returns for the economic sectors of the Dow are employed. The performance of the models is evaluated by comparing the variance explained by the first four principal components of the conditional volatilities, the amount of unexplained serial correlation in the squared standardized residuals and the degree of temporal dependence left in the multivariate residuals.

The main findings are that all the PVCF models perform rather well in terms of explained variance, residual serial correlation and temporal dependence. Only a small number of variance factors are needed to describe the comovements in the Dow Jones' volatilities. Such latent volatility factors are related both to the market as a whole and to the economic sectors, which may respond differently to the same news. However, no model seems to clearly outperform the others, and further research is needed to develop new methodologies to characterize and test PVCF.

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