



Contents lists available at ScienceDirect

Journal of Monetary Economics

journal homepage: www.elsevier.com/locate/jme

The intertemporal capital asset pricing model with dynamic conditional correlations[☆]

Turan G. Bali^{a,*}, Robert F. Engle^{b,1}

^a Department of Economics and Finance, Zicklin School of Business, Baruch College, One Bernard Baruch Way, Box 10-225, New York, NY 10010, USA

^b New York University, Stern School of Business, 44 West Fourth Street, Suite 9-62, New York, NY 10012, USA

ARTICLE INFO

Article history:

Received 25 November 2009

Received in revised form

21 March 2010

Accepted 25 March 2010

Available online 7 April 2010

Keywords:

ICAPM

Dynamic conditional correlation

ARCH

Risk aversion

Risk factors

ABSTRACT

The intertemporal capital asset pricing model of Merton (1973) is examined using the dynamic conditional correlation (DCC) model of Engle (2002). The mean-reverting DCC model is used to estimate a stock's (portfolio's) conditional covariance with the market and test whether the conditional covariance predicts time-variation in the stock's (portfolio's) expected return. The risk-aversion coefficient, restricted to be the same across assets in panel regression, is estimated to be between two and four and highly significant. The risk premium induced by the conditional covariation of assets with the market portfolio remains positive and significant after controlling for risk premia induced by conditional covariation with macroeconomic, financial, and volatility factors.

© 2010 Elsevier B.V. All rights reserved.

1. Introduction

Merton (1973) introduces an intertemporal capital asset pricing model (ICAPM) in which an asset's expected return depends on its covariance with the market portfolio and with state variables that proxy for changes in investment opportunity set. A large number of studies test the significance of an intertemporal relation between expected return and risk in the aggregate stock market. However, even the existence of a positive risk-return tradeoff for market indices has not been universally found in the existing literature. Due to the fact that the conditional mean and volatility of stock market returns are not observable, different approaches and specifications used by previous studies in estimating the two conditional moments are largely responsible for the conflicting empirical evidence.

Our study extends time-series tests of the ICAPM to many risky assets. The prediction of Merton (1980) that expected returns should be related to conditional risk applies not only to the market portfolio but also to individual stocks and stock

[☆] We are grateful to the Editor (Robert King), the Associate Editor (Urban Jermann), and an anonymous referee for their extremely helpful comments and suggestions. We thank Yakov Amihud, Andrew Ang, Tim Bollerslev, Stephen Brown, Frank Diebold, Michael Fleming, Xavier Gabaix, Christian Lundblad, Anthony Lynch, Hui Guo, Matthew Richardson, Stijn Van Nieuwerburgh, Robert Whitelaw, and seminar participants at the 2009 American Finance Association meetings, 2008 Inaugural Conference of the Society for Financial Econometrics, 2008 Financial Management Association meetings, 2008 China International Conference in Finance, and at Fordham University, New York University, Oxford University, Rutgers University, and the University of Iowa for useful discussions. We also thank Long Chen, Kenneth French, and Lu Zhang for making a large amount of historical data publicly available in their online data library.

* Corresponding author. Tel.: +1 646 312 3506; fax: +1 646 312 3451.

E-mail addresses: turan.bali@baruch.cuny.edu (T.G. Bali), rengle@stern.nyu.edu (R.F. Engle).

¹ Tel.: +1 212 998 0710; fax: +1 212 995 4220.

portfolios. Expected returns for any stock should vary through time with the stock's conditional covariance with the market portfolio as well as the hedging demands. To be internally consistent, the relation should be the same for all stocks, i.e., the predictive slope on the conditional covariance represents the average relative risk aversion of market investors. We exploit this cross-sectional consistency condition and estimate the common time-series relation across 30 stocks in the Dow Jones Industrial Average.

Using daily data from July 1986 to June 2009, the mean-reverting dynamic conditional correlation (DCC) model of Engle (2002) is estimated to generate the time-varying conditional covariances between daily excess returns on each stock and the market portfolio. Then, a system of time-series regressions of the stocks' excess returns on their conditional covariances with the market is estimated, while constraining all regressions to have the same slope coefficient. Our estimation based on Dow 30 stocks and alternative measures of the market portfolio generates positive and highly significant risk aversion coefficients, with magnitudes between two and four. The identified positive risk-return tradeoff is robust to different market portfolios, different sample periods, alternative specifications of the conditional mean and covariance processes, different data sets including stocks in the S&P 100 and S&P 500 indices, and including a wide variety of state variables that proxy for the intertemporal hedging demand component of the ICAPM. The significance of an intertemporal relation is investigated based on the value-weighted size, book-to-market, momentum, industry, investment-to-assets, and return-on-assets portfolios. First, using daily data from January 1972 to June 2009, the DCC-based conditional covariances between daily excess returns on equity portfolios and the market portfolio are estimated. Then, the predictive panel regressions of the portfolios' excess returns are run on their conditional covariances with the market, while imposing the same risk aversion coefficient across portfolios. The results from various equity portfolios provide evidence for a significantly positive link between risk and return, with economically sensible estimates of risk aversion coefficients.

When the investment opportunity is stochastic, investors adjust their investment to hedge against unfavorable shifts in the investment opportunity set and achieve intertemporal consumption smoothing. Hence, covariations with state of the investment opportunity induce additional risk premium on an asset. A series of macroeconomic, financial, and volatility factors are identified and then empirical tests are conducted to determine if their conditional covariances with individual stocks (and stock portfolios) induce additional risk premia.

To explore how macroeconomic variables vary with the investment opportunity and test whether covariations of stocks (portfolios) with them induce additional risk premia, the conditional covariances of these variables with daily excess returns on each stock (portfolio) is estimated first and then empirical analyses are performed to detect how the stocks' (portfolios') excess returns respond to their conditional covariances with macroeconomic factors. Because of data availability at daily frequency, the federal funds rate, default and term spreads are used as potential factors that may affect the investment opportunity set. The parameter estimates show that incorporating the covariances of stock returns with the aforementioned macroeconomic variables does not alter the magnitude and statistical significance of the relative risk aversion coefficient. The common slope on the market covariance remains positive and highly significant. The results also indicate that the slope coefficients on the conditional covariances with the changes in default and term spreads are statistically significant, implying that unexpected news in macroeconomic variables do contain systematic risks rewarded in the stock market.

Fama and French (1992, 1993) provide evidence on the significance of size and book-to-market variables in predicting the cross-sectional and time-series variation in stock and portfolio returns. Jegadeesh and Titman (1993) and Carhart (1997) present evidence on the significance of past returns (or momentum) in predicting the cross-sectional and time-series variation in future returns on individual stocks and portfolios. Chen and Zhang (2009) introduce two new factors, the investment-to-assets and the return-on-assets, that provide a better characterization of the cross-section of expected stock returns. We examine whether the size (*SMB*), book-to-market (*HML*), and momentum (*MOM*) factors of Fama-French (1993) and Carhart (1997) as well as the investment-to-assets (*IA*) and the return-on-assets (*ROA*) factors of Chen and Zhang (2009) move closely with investment opportunities and whether covariations of individual stocks (portfolios) with these factors induce additional risk premia on stocks (portfolios). Estimation shows that the covariances of daily stock returns with the *HML*, *IA*, and *ROA* factors generate significantly positive slope coefficients. Hence, an increase in a stock's (portfolio's) covariance with *HML*, *IA*, and *ROA* predicts a higher excess return on the stock (portfolio). The results also indicate that the covariances of stocks (portfolios) with the *SMB* and *MOM* factors do not have robust, significant predictive power for future returns on individual stocks (portfolios).

Following Campbell (1993, 1996), it is assumed that investors want to hedge against the changes in the forecasts of future market volatilities. In this paper, implied volatility from the S&P 100 index options is used to test whether stocks that have higher correlation with the changes in future market volatility yield lower expected return. The results indicate that the risk premium induced by the conditional covariation of individual stocks (portfolios) with the market portfolio remains economically and statistically significant after controlling for risk premia induced by conditional covariation with the changes in implied volatility. The results also provide evidence for a significantly negative relation between expected return and volatility risk, i.e., assets with higher association with the changes in future market volatility give lower expected return.

The paper is organized as follows: Section 2 briefly discusses earlier studies on the intertemporal relation between expected return and risk. Section 3 describes the data and estimation methodology. Section 4 presents the empirical results. Section 5 concludes.

2. Literature on the risk-return tradeoff

Dynamic asset pricing models starting with Merton's (1973) ICAPM provide a theoretical framework that gives a positive equilibrium relation between the conditional first and second moments of excess returns on the market portfolio. However, Abel (1988), Backus and Gregory (1993), and Genotte and Marsh (1993) develop models in which a negative relation between expected return and volatility is consistent with equilibrium. Similarly, empirical studies are still not in agreement on the direction of a time-series relation between expected return and risk.

Many studies fail to identify a robust and significant intertemporal relation between risk and return on the aggregate stock market portfolio. French et al. (1987) find that the risk-return coefficient is not significantly different from zero when they use past daily returns to estimate the monthly conditional variance. Follow-up studies by Baillie and DeGennaro (1990), Campbell and Hentschel (1992), Glosten et al. (1993), Harrison and Zhang (1999), and Bollerslev and Zhou (2006) rely on the GARCH-in-mean and realized volatility models that provide no evidence for a robust, significant link between expected return and risk on the aggregate market portfolio.

Several studies even find that the intertemporal relation between risk and return is negative. Examples include Campbell (1987), Breen et al. (1989), Nelson (1991), Glosten et al. (1993), Whitelaw (1994), Harvey (2001), and Brandt and Kang (2004). Some studies do provide evidence supporting a positive and significant link between expected return and risk (e.g., Bollerslev et al., 1988; Scruggs, 1998; Ghysels et al., 2005; Guo and Whitelaw, 2006; Lundblad, 2007; Bali, 2008).

3. The intertemporal relation between expected return and risk

Merton's (1973) ICAPM implies the following equilibrium relation between risk and return:

$$\mu_{t+1} - r_{f,t} = A \cdot Cov_t(r_{t+1}, r_{m,t+1}) + Cov_t(r_{t+1}, x_{t+1}) \cdot B, \quad (1)$$

where $r_{f,t}$ is the risk free rate, $\mu_{t+1} = E(r_{t+1})$ is the $n \times 1$ vector conditional mean of stock returns r_{t+1} at time $t+1$, $r_{m,t+1}$ is the market return, and x_{t+1} is a vector of k state variables that shift the investment opportunity set. $Cov_t(r_{t+1}, r_{m,t+1})$ is the time- t expected conditional covariance between r_{t+1} and $r_{m,t+1}$, i.e., the covariances are conditional on information available at the time the assets are evaluated. Important restrictions implied by the theory are the fact that intercepts in this equation are zero and that the slope coefficient A is a scalar that is appropriate for all assets and B is a $k \times 1$ vector that prices all assets. This representation is equivalent to non-existence of arbitrage when the pricing kernel is a linear function of the aggregate market return. Additional factors could be added and will be examined in the empirical work. There is no restriction in the theory that the parameters A and B are time invariant although this is commonly applied and is reasonable when the state variables are included to measure shifts in investment opportunity sets.

In the original Merton model, the parameters of the system and the covariances were all interpreted as constant but the ability to model time variation in covariances makes it natural to include these directly in the analysis. The empirical literature has generally recognized that the parameters may be time varying but not in a parametric fashion. In principle, if the covariances are stochastic, they would represent additional sources of variation in the investment opportunity set and potential hedging demand terms.

The expression in (1) can now be written as

$$V_t \begin{pmatrix} r_{t+1} \\ r_{m,t+1} \\ x_{t+1} \end{pmatrix} = \begin{pmatrix} H_{r,r,t+1} H_{r,m,t+1} H_{r,x,t+1} \\ H_{m,r,t+1} H_{m,m,t+1} H_{m,x,t+1} \\ H_{x,r,t+1} H_{x,m,t+1} H_{x,x,t+1} \end{pmatrix}, \quad (2)$$

$$E_t(r_{t+1}) = r_{f,t} + A \cdot H_{r,m,t+1} + H_{r,x,t+1} \cdot B,$$

$$E_t(r_{m,t+1}) = r_{f,t} + A \cdot H_{m,m,t+1} + H_{m,x,t+1} \cdot B. \quad (3)$$

If normality is assumed, then the first two moments are sufficient to define the distribution; hence, Eqs. (2) and (3) define the process regardless of how many assets are being priced.

Various parameterizations of the covariance matrix (V_t) have been used in the literature. In this paper, several versions of the DCC model are employed as this is very stable and parsimonious and is manageable for large numbers of assets.² Furthermore, it is clear from (3) that the full dynamic covariance matrix of returns does not enter the pricing equation. Thus a simplified estimation approach is employed for most of the analyses in this paper. In particular, the covariances $H_{r,m,t+1}$ and $H_{r,x,t+1}$ can be estimated first and used in a seemingly unrelated regression estimate of returns in (3). This is described in more detail in Section 3.2.

In each case, the hypothesis that A equals zero and the hypothesis that the individual intercepts are jointly zero are tested. For some state variables, B is found to be significantly different from zero, but the significance of state variables does not affect the estimates of A . A notable point is that, volatilities; however, they are measured, are highly significant

² The covariance matrix (V_t) is more explicitly defined in Section 3.2. The maximum likelihood estimation of V_t and the DCC model is provided in the online supplement.

but still do not alter our estimates for A . Allowing the state variables x_{t+1} to enter (3) directly does not change the estimates appreciably.

3.1. Data

The latest stock composition of the Dow Jones Industrial Average is used. The ticker symbols and company names are presented in Appendix A of the online supplement. In our empirical analyses, daily excess returns on Dow 30 stocks are used for the longest common sample period from July 10, 1986 to June 30, 2009, yielding a total of 5795 daily observations. Appendix B of the online supplement reports the mean, median, maximum, minimum, and standard deviation of daily excess returns on Dow 30 stocks.

For the market portfolio, five different stock market indices are used: (1) the value-weighted NYSE/AMEX/NASDAQ index, also known as the value-weighted index of the Center for Research in Security Prices (CRSP), can be viewed as the broadest possible stock market index used in earlier studies, (2) New York Stock Exchange (NYSE) index, (3) Standard and Poor's 500 (S&P 500) index, (4) Standard and Poor's 100 (S&P 100) index, and (5) Dow Jones Industrial Average (DJIA) can be viewed as the smallest possible stock market index used in earlier studies. Our findings from alternative measures of the market portfolio are very similar. To save space, results are reported only for the CRSP value-weighted index. The findings from alternative market portfolios are provided in the online supplement and also in Bali and Engle (2007).

In addition to Dow 30 stocks, the value-weighted decile portfolios of size, book-to-market, momentum, industry, investment-to-assets (IA), and return-on-assets (ROA) are used for the longest common sample from January 3, 1972 to June 30, 2009 (9462 daily observations). The daily returns on the size, book-to-market, momentum, and industry portfolios are obtained from Kenneth French's online data library. The daily returns on the IA and ROA portfolios are obtained from Long Chen's and Lu Zhang's online data library.

Appendix C of the online supplement presents the average daily excess returns on equity portfolios and the annualized return differences between the extreme portfolios. The first column shows that there is no significant size premium for the value-weighted portfolios. Specifically, the average return difference between the small and big size portfolios is only -0.38% per annum and statistically insignificant. The results indicate significant value premium as the annual average return on the value portfolio is 6.68% higher than the annual average return on the growth portfolio. The average return difference between the winner and loser portfolios is 15.88% per year with a Newey–West t -statistic of 3.77 , implying economically and statistically significant momentum profits. There is no significant return difference between the extreme industry portfolios. Specifically, the annualized return difference between the highest-return (Enrgy) and lowest-return (Durbl) industry portfolios is 5.72% per annum with a Newey–West t -statistic of 1.58 . Similar to the findings of Chen and Zhang (2009), stocks in the low IA portfolio generates 8.92% higher return than those in the high IA portfolio and this return difference is highly significant with a t -statistic of 4.44 . The high and low ROA portfolios generate even higher spread of 13.12% per annum with a t -statistic of 4.11 .

For state variables, the commonly used macroeconomic variables (fed funds rate, default spread, and term spread), financial factors (size, book-to-market, momentum, investment, and ROA), and volatility measures (options' implied, GARCH, and range) are considered.

3.1.1. Macroeconomic variables

Daily data on the federal funds rate, 3-month Treasury bill, 10-year Treasury bond yields, BAA-rated and AAA-rated corporate bond yields are obtained from the H.15 database of the Federal Reserve Board. In addition to the fed funds rate, the term and default spreads are used as control variables. The term spread is calculated as the difference between the yields on the 10-year Treasury bond and the 3-month Treasury bill. The default spread is computed as the difference between the yields on the BAA-rated and AAA-rated corporate bonds.

3.1.2. Size, book-to-market, momentum, investment, and ROA factors

The SMB (small minus big) factor is the difference between the returns on the portfolio of small size stocks and the returns on the portfolio of big size stocks. The average return on the SMB factor is positive but economically and statistically insignificant; 0.81% per annum with a t -statistic of 0.52 . The HML (high minus low) factor is the difference between the returns on the portfolio of high book-to-market stocks and the returns on the portfolio of low book-to-market stocks. The average return on the HML factor is positive and significant; 5.21% per annum with a t -statistic of 3.27 . The MOM (winner minus loser) factor is the difference between the returns on the portfolio of stocks with higher past 2- to 12-month cumulative returns (winners) and the returns on the portfolio of stocks with lower past 2- to 12-month cumulative returns (losers). The average return on the MOM factor is large and highly significant; 8.66% per annum with a t -statistic of 3.54 .

The IA factor is the difference (low-minus-high I/A) between the average daily returns on the two low- I/A portfolios and the average daily returns on the two high- I/A portfolios. The average return on the IA factor is economically and statistically significant; 4.96% per annum with a t -statistic of 4.38 . The ROA factor is the difference (high-minus-low ROA) between the average daily returns on the two high- ROA portfolios and the average daily returns on the two low- ROA

portfolios. The average return on the ROA factor is the largest among all these financial factors; 11.04% per annum with a t -statistic of 5.06.

3.1.3. Alternative measures of market volatility

Implied volatilities are considered to be the market's forecast of the volatility of the underlying asset of an option. Specifically, the Chicago Board Options Exchange (CBOE)'s VXO implied volatility index provides investors with up-to-the-minute market estimates of expected volatility by using real-time S&P 100 index option bid/ask quotes. The VXO is a weighted index of American implied volatilities calculated from eight near-the-money, near-to-expiry, S&P 100 call and put options based on the Black-Scholes (1973) pricing formula.

As an alternative to the VXO index, we could have used the newer VIX index, which was introduced by the CBOE on September 22, 2003. The VIX is obtained from the European style S&P 500 index option prices and incorporates information from the volatility skew by using a wider range of strike prices than just at-the-money series. However, the daily data on VIX starts from January 2, 1990, which does not cover our full sample period (7/10/1986–6/30/2009). Hence, the daily data on VXO are used starting from January 2, 1986 and spanning the full sample period of Dow 30 stocks.

In addition to the VIX and VXO implied volatility indices, the conditional variance of daily excess returns on the S&P 500 index is estimated using a GARCH(1,1) model and then the DCC-based conditional covariances are generated for daily excess returns on individual stocks (portfolios) and the change in daily GARCH volatility. Our objective is to test whether unexpected news in market volatility is priced in the stock market and then to check robustness of risk-aversion coefficient after controlling for risk premium induced by the conditional covariation of stocks (portfolios) with the GARCH volatility of the market portfolio.

As a further robustness check, the range volatility that utilizes information in the high frequency intraday data is used:

$$\text{Range}_{m,t} = \ln(P_{m,t}^{\max}) - \ln(P_{m,t}^{\min}), \quad (4)$$

where $P_{m,t}^{\max}$ and $P_{m,t}^{\min}$ are the highest and lowest stock market index levels on day t . In our empirical analysis, the maximum and minimum values of the S&P 500 index are used over a sampling interval of one day. As pointed out by Alizadeh et al. (2002) and Brandt and Diebold (2006), Eq. (4) can be viewed as a measure of daily standard deviation of the market. Note that our findings are insensitive to the choice of volatility measures and available in the online supplement. To save space results are reported only for the implied volatility.

3.2. Estimating time-varying conditional covariances

The DCC model of Engle (2002) parameterizes the volatilities and correlations separately. The general specification is

$$y_{t+1} \equiv \begin{pmatrix} r_{t+1} \\ r_{t+1}^m \\ x_{t+1} \end{pmatrix} = \alpha_0 + \alpha_1 y_t + \mu_t + \varepsilon_{t+1}, \quad V_t(\varepsilon_{t+1}) = D_{t+1} \rho_{t+1} D_{t+1}, \quad (5)$$

where the mean is given by Eq. (1) for asset returns, and D is the diagonal matrix of conditional standard deviations given by

$$D_{t+1}^2 = \beta_0 + \beta_1 y_t^2 + \beta_2 D_t^2, \quad \alpha_0, \alpha_1, \alpha_2, \beta_0, \beta_1, \beta_2, D_t \text{ diagonal}. \quad (6)$$

The correlations are given by

$$\rho_t = \text{diag}(Q_t)^{-1/2} Q_t \text{diag}(Q_t)^{-1/2}, \quad (7)$$

where

$$Q_{t+1} = \bar{\rho} + a_1 \cdot (u_t u_t' - \bar{\rho}) + a_2 \cdot (Q_t - \bar{\rho}), \quad u_t = D_t^{-1} \varepsilon_t, \quad \bar{\rho} = \frac{1}{T} \sum_{t=1}^T u_t u_t'. \quad (8)$$

In many applications, a_1 and a_2 are considered to be scalars as in Engle (2002) and Engle (2009a, b), however we consider a more flexible specification where these are symmetric positive definite matrices and multiplication is element by element or Hadamard. This is shown by Ding and Engle (2001) to imply positive definiteness of the matrix Q and consequently of the correlation matrix. In this way each return may have different parameters. Our estimation strategy does not enforce positive definiteness as it consists of a series of bivariate estimations, but it does insure that all correlations used in estimation lie in the interval $(-1, 1)$.

The likelihood function for this problem is constructed assuming conditional normality of the dependent variables. As is generally true for multivariate GARCH models, this estimator is consistent even if the normality assumption is invalid as long as the first two moment equations are correctly specified (see Bollerslev and Wooldridge (1992) for a proof).

Our estimation approach proceeds in steps:

- (1) Any autoregressive elements in returns are taken out and univariate GARCH models are estimated for all returns and state variables.

- (2) Standardized returns are constructed and the bivariate DCC estimates of the correlations are computed for each stock (portfolio) and the market and for each stock (portfolio) and the state variables using the bivariate likelihood function.
- (3) The expected return equation is estimated as a panel with the conditional covariances as regressors. The error covariance matrix is specified as seemingly unrelated regression (SUR). A weighted least squares alternative divides each equation by its estimated conditional standard deviation before estimating the panel by SUR.

In one case the entire system is jointly estimated and similar results are obtained.³

Although not reported in the paper to save space, both parameters of the mean-reverting DCC model ($0 < a_1, a_2 < 1$) are estimated to be positive, less than one, and highly significant. Similar to the findings of Engle (2002), the magnitude of a_1 is small, in the range of 0.01 to 0.06, whereas a_2 is found to be large, ranging from 0.93 to 0.99. The persistence of the conditional correlations of each stock (portfolio) with the market portfolio is measured by the sum of a_1 and a_2 . For all datasets used in the paper, the estimated value of $(a_1 + a_2)$ is less than one, implying mean reversion in the conditional correlation estimates. Although not presented in the paper to preserve space, the conditional correlations exhibit significant time variation for all stocks and portfolios and the correlations are pulled back to some long-run average level over time, indicating strong mean reversion. A common observation is that when the level of conditional correlation is high, mean reversion tends to cause it to have a negative drift, and when it is low, mean reversion tends to cause it to have a positive drift.

To test whether the mean-reverting DCC model generates reasonable conditional covariance estimates, the equal-weighted and price-weighted averages are computed for the conditional covariances of Dow 30 stocks with the market portfolio. Then, the weighted average conditional covariances are compared with the benchmark of the conditional market variance. The weighted-average covariances are found to be in the same range as the conditional variance of the market portfolio. The two series move very closely together. Specifically, the sample correlation between the equal-weighted average covariance and the market variance is 0.9931 and the sample correlation between the price-weighted average covariance and the market variance is 0.9932. The affinity in magnitudes and time-series fluctuations between the weighted average covariances and market variance provides evidence for reasonable conditional variance and covariance estimates from the mean-reverting DCC model. A set of DCC parameter estimates as well as the time-series plot of the conditional correlations are given in the online supplement.

3.3. The role of the DCC model in testing ICAPM

Asset allocation and risk management decisions depend on correlations, but a large number of correlations is often required in practice. Optimal portfolio selection requires a forecast of the covariance matrix of asset returns. Similarly, the calculation of the standard deviation, value-at-risk, or expected shortfall of a portfolio requires a covariance matrix of all the assets in the portfolio. This necessitates estimation and prediction of large covariance matrices. Simple methods such as rolling historical correlations and exponential smoothing are widely used, but they do not well capture the time-series variation in correlations. More complex methods, such as varieties of multivariate GARCH models have been proposed and investigated in the econometrics literature (e.g., Bollerslev et al., 1988; Bollerslev, 1990; Engle et al., 1990; Ding and Engle, 2001). In earlier studies, the authors generally use two or three assets, not more than five assets are considered in previous studies, despite the apparent need for bigger correlation matrices.

The DCC model of Engle (2002) has clear computational advantages over multivariate GARCH models in that the number of parameters to be estimated in the correlation process is independent of the number of series to be correlated.⁴ Therefore, potentially very large correlation matrices can be estimated with the DCC approach. Bollerslev et al. (1988) provide the first multivariate test of the conditional CAPM. With their multivariate GARCH-in-mean model, Bollerslev et al. (1988) were able to use three assets (not more than three) when investigating the significance of an intertemporal relation.⁵ They obtained significant results only when they allowed intercepts (alphas) to vary across assets. Otherwise, they found positive but statistically insignificant risk aversion coefficients.

In this paper, we have the flexibility to increase the number of assets to 10 (equity portfolios) and to 30, 100, and even 500 (stocks in the Dow Jones, S&P 100, and S&P 500 indices) because the DCC approach allows us to estimate a large number of conditional covariances. By pooling a large number of time-series and cross-section together, we find that the DCC-based conditional covariance estimates predict the time-series and cross-sectional variation in stock returns and they generate significant and reasonable risk premium. The power of our methodology is coming from the fact that the first-stage DCC estimation provides very accurate characterization of time-series variation in conditional covariances and the second-stage SUR regressions well capture the cross-sectional correlations among assets. The robust, significant and economically sensible estimates of risk aversion coefficients highlight the added benefits of using the DCC-based

³ The panel estimation methodology with SUR takes into account heteroskedasticity and autocorrelation as well as contemporaneous cross-correlations in the error terms.

⁴ The DCC model, which parameterizes the conditional correlations directly, is naturally estimated in two steps—a series of univariate GARCH estimates and the correlation estimate.

⁵ Testing the conditional CAPM with a large number of assets is not possible within the multivariate GARCH-in-framework introduced by Bollerslev et al. (1988) and follow-up related studies.

conditional measures of risk and simultaneously maintaining the large number of assets in cross-sectional testing of the ICAPM.

4. Empirical results

Firstly, the estimation results on the risk-return tradeoff are presented assuming zero intertemporal hedging demand. Secondly, our main findings are looked into whether they are driven by a particular set of individual stocks. Thirdly, some evidence from the one-step estimation of the ICAPM with DCC is provided. Fourthly, the robustness of our main findings is checked after controlling for unexpected news in macroeconomic variables and market volatility. Finally, the intertemporal relation is estimated by including additional risk premia induced by the conditional covariation of stocks (portfolios) with various macroeconomic, financial, and volatility factors.

4.1. Risk-return tradeoff without intertemporal hedging demand

Table 1 reports the common slope estimates (A) along with the t -statistics from the following system of equations:

$$R_{i,t+1} = C_i + A \cdot \sigma_{im,t+1} + e_{i,t+1}, \quad (9)$$

where $R_{i,t+1}$ denotes the daily excess return on stock i (or portfolio i) at time $t+1$, $R_{m,t+1}$ denotes the daily excess return on the market portfolio at time $t+1$, and $\sigma_{im,t+1}$ is the time- t expected conditional covariance between $R_{i,t+1}$ and $R_{m,t+1}$. C_i is the intercept for asset i and A is the common slope coefficient. Estimation is based on daily returns on the 10 value-weighted size, book-to-market, momentum, industry, investment-to-assets (IA), and return-on-assets (ROA) portfolios for the sample period of January 3, 1972 to June 30, 2009. The last row of Table 1 presents results for Dow 30 stocks for the sample period of July 10, 1986 to June 30, 2009. The market portfolio is proxied by the value-weighted NYSE/AMEX/NASDAQ index.

Table 1 shows that the risk-return coefficient on $\sigma_{im,t+1}$ is estimated to be positive and highly significant with the t -statistics ranging from 3.82 to 8.68. The common slope estimates are stable across different test assets, between 1.59 and 3.32. Based on the relative risk aversion interpretation, the magnitudes of these estimates are economically sensible as well.

To gain some insight about the economic significance of risk-return coefficients, Eq. (9) can be rewritten in a static CAPM framework:

$$E(R_i) = C_i + (A \cdot \sigma_m^2) \cdot \left(\frac{\sigma_{im}}{\sigma_m^2} \right) = C_i + (A \cdot \sigma_m^2) \cdot \beta_i = C_i + E(R_m) \cdot \beta_i, \quad (10)$$

where expected excess return on stock i , $E(R_i)$, is a linear function of market beta, β_i , and $(A \cdot \sigma_m^2)$ is a measure of expected excess return on the market portfolio, i.e., $E(R_m) = A \cdot \sigma_m^2$.

For the long sample of January 1972 to June 2009, the standard deviation of excess returns on the market portfolio is 1.04% per day, implying that the daily variance of the market portfolio, σ_m^2 , is about 0.0001. As reported in Table 1, the common slope (A) is estimated in the range of 1.59 and 3.32, which corresponds to $A \cdot \sigma_m^2 = 4.01\%$ and 8.37% annualized expected market risk premium assuming 252 trading days in a year. For the short sample of July 1986 to June 2009, the standard deviation of excess returns on the market portfolio is 1.15% per day, implying that the daily σ_m^2 is about 0.00013.

Table 1

Risk-return tradeoff without intertemporal hedging demand.

| Test assets | $\sigma_{im,t+1}$ | Wald |
|----------------|-------------------|--------------|
| Size | 1.8558 (5.07) | 13.29 [0.21] |
| Book-to-market | 2.0546 (5.29) | 15.73[0.11] |
| Momentum | 3.3187 (8.68) | 36.71 [0.00] |
| Industry | 1.8532 (4.86) | 9.53 [0.48] |
| IA | 1.5904 (3.82) | 30.35 [0.00] |
| ROA | 2.2464 (6.09) | 30.80 [0.00] |
| Dow 30 stocks | 2.2139 (7.53) | 18.17 [0.96] |

Entries report the common slope estimates (A) and their t -statistics (in parentheses) from the following system of equations,

$$R_{i,t+1} = C_i + A\sigma_{im,t+1} + e_{i,t+1}, \quad i = 1, 2, \dots, n,$$

where $R_{i,t+1}$ denotes the daily excess return on stock i (or portfolio i) at time $t+1$, $R_{m,t+1}$ denotes the daily excess return on the market portfolio at time $t+1$, and $\sigma_{im,t+1}$ is the time- t expected conditional covariance between $R_{i,t+1}$ and $R_{m,t+1}$. C_i is the intercept for asset i and A is the common slope coefficient. Estimation is based on daily returns on the 10 value-weighted size, book-to-market, momentum, industry, investment-to-assets (ia), return-on-assets (ROA), and the value-weighted NYSE/AMEX/NASDAQ market portfolio for the sample period of January 3, 1972 to June 30, 2009. The last row presents results for Dow 30 stocks for the sample period of July 10, 1986 to June 30, 2009. The first column shows the common slope coefficients (A) on $\sigma_{im,t+1}$ and their t -statistics in parentheses. The last column reports the Wald statistics and the p -values in square brackets from testing the joint hypothesis of all intercepts equal zero, $H_0: C_i = 0$.

As reported in the last row of Table 1, A is estimated to be 2.21 for Dow 30 stocks, which corresponds to $A \cdot \sigma_m^2 = 7.24\%$ annualized expected market risk premium.

In estimating the system of time-series relations, the intercepts are allowed to differ across stocks (portfolios). These intercepts (or conditional alphas) capture the daily abnormal returns on each stock (portfolio) that cannot be explained by the conditional covariances with the market portfolio. The last column of Table 1 reports the Wald statistics and the p -values in square brackets from testing the joint hypothesis of all intercepts equal zero; $H_0: C_i=0$. The Wald statistic is very small for Dow 30 stocks; 18.17 with a p -value of 0.96 for 30 degrees of freedom, indicating that the conditional covariances of Dow 30 stocks with the market portfolio have significant predictive power for the time-series and cross-sectional variation in expected returns, i.e., the ICAPM with DCC holds.

The Wald statistics for the size, book-to-market, and industry portfolios are, respectively, 13.29, 15.73, and 9.53 with the corresponding p -values of 0.21, 0.11, and 0.48. The significantly positive risk aversion coefficients and the insignificant Wald statistics indicate that the DCC-based conditional covariances capture the time-series and cross-sectional variation in returns on size, book-to-market, and industry portfolios because the two essential tests of the ICAPM are passed: (i) positive common slope ($A > 0$) and (ii) zero intercepts ($C_i=0$).

Another notable point in Table 1 is that the conditional CAPM does not hold for the momentum, IA, and ROA portfolios. Although there is a positive and significant link between expected return and risk on these portfolios ($A > 0$), the Wald statistics from testing the joint hypothesis of zero intercepts are very high, in the range of 30.35 to 36.71, with zero p -values. Overall, the GARCH methodology with DCC resurrects the conditional CAPM for individual stocks and stock portfolios with relatively small cross-sectional spreads. However, the tests reject the conditional CAPM for portfolios with significantly large cross-sectional spreads between extreme portfolios [momentum (winner vs. loser); IA (low IA vs. high IA); and ROA (high ROA vs. low ROA)].

4.2. Stocks in the S&P 500 and S&P 100 indices

To determine whether the results are driven by a particular data set, our main findings are replicated with individual stocks in the S&P 500 and S&P 100 indices. First, the DCC based conditional covariances of each stock in the S&P 500 index with the market portfolio are estimated and then the SUR panel regression is run. The common slope on $\sigma_{im,t+1}$ is found to be 2.67 with a t -statistic of 6.21, indicating a positive and highly significant relation between expected return and risk on S&P 500 stocks. We also test the joint hypothesis of all intercepts equal zero and the Wald statistic has a p -value of 0.95, implying that the conditional alphas are jointly zero, i.e., the ICAPM with DCC holds. For the sample of S&P 100 stocks, the risk aversion coefficient is estimated to be 4.58 with a t -statistic of 7.50, giving a significantly positive link between expected return and risk on S&P 100 stocks. The Wald statistic from testing the joint hypothesis of zero intercepts has a p -value of 0.91 for the S&P 100 sample. These results provide evidence that the significantly positive relation between risk and return is robust across different stock samples.

4.3. One-step estimation with Dow 30 stocks

The risk aversion coefficient has so far been estimated in two steps; first obtain the conditional covariances with DCC and then use the covariance estimates in the panel regression with a common slope coefficient. Some readers might be concerned that the covariance matrices implied by the DCC model were not used in estimating risk premia or in computing their standard errors. A common worry in testing asset pricing models is that time-varying covariances are measured with error. Since this is the first study that utilizes DCC based conditional covariances in examining an asset pricing model, the reader is not aware of the significance of measurement errors in covariances.

As pointed out by earlier studies, estimating multivariate GARCH-in-mean models with time-varying conditional correlations is an extremely difficult task, especially if the number of assets gets bigger. Early work on time-varying covariances in large dimensions was carried out by Bollerslev (1990) in his constant correlation model, where the volatilities of each asset were allowed to vary through time but the correlations were time invariant (see Tse (2000) and Engle (2009a) for a review of this topic). Recently, the DECO model of Engle and Kelly (2007) and the MacGyver estimation method of Engle (2009b) deal with the computation of correlations for a large number of assets with an assumption that the correlation amongst assets changes through time but is constant across the cross-section of assets.⁶ Note that estimating time-varying correlations based on a multivariate GARCH model with a constant mean is easier than estimating time-varying correlations based on a multivariate GARCH-in-mean model. In this paper, the time-varying conditional correlations as well as the parameters of time-varying conditional mean are estimated in one step using a multivariate GARCH-in-mean framework.

To ease the parameter convergence, correlation targeting is used assuming that the time-varying correlations mean reverts to the sample correlations. To reduce the overall time of maximizing the conditional log-likelihood, all pairs of the

⁶ An alternative method was suggested by Engle (2009b) where he fit many pairs of bivariate estimators, governed by simple dynamics, and then took a median of these estimators. This method is known as the MacGyver estimation strategy. Engle et al. (2008) introduce a new estimation methodology that has some similarities to the MacGyver strategy, but it is more efficient.

bivariate GARCH-in-mean model are estimated first and then the median values of A , a_1 and a_2 are used as starting values along with the bivariate GARCH-in-mean estimates of variance parameters (β_0 , β_1 , β_2). Even after going through these steps to increase the speed of parameter convergence, it takes very long time to obtain the full set of parameters in the multivariate GARCH-in-mean model with 30 stocks and 5,795 time-series observations.

The common slope and the intercepts are estimated using daily returns on the Dow 30 stocks for the period July 10, 1986 to June 30, 2009. The market portfolio is proxied by the value-weighted CRSP index. The risk-return coefficient (A) on $\sigma_{im,t+1}$ is estimated to be 2.86 with a t -statistic of 5.67. The magnitude and statistical significance of the common slope turns out to be similar to our earlier findings from the two-step estimation. As shown in Table 1, the risk aversion parameter (A) is estimated to be 2.21 with t -stat.=7.53 from the SUR panel regression. The joint hypothesis of all intercepts equal zero is tested in the multivariate GARCH-in-mean model. The Wald statistic has a p -value of 0.71. Overall, the one-step and two-step estimation results provide similar evidence such that there is a significantly positive relation between risk and return on Dow stocks and the abnormal daily returns are economically and statistically insignificant.

4.4. Controlling for unexpected news in macroeconomic variables

Merton (1973) indicates that any variable that affects future investment opportunities could be a priced risk factor in equilibrium. Ross (1976) further documents that securities affected by such systematic risk factors should earn risk premia in a risk-averse economy. Macroeconomic variables are excellent candidates for these systematic risk factors because innovations or unexpected changes in macro variables can generate global impact on firms' fundamentals, such as their cash flows, risk-adjusted discount factors, and investment opportunities.

To determine whether unexpected news in macroeconomic factors can influence time-series variation in stock returns and hence may affect the risk-return tradeoff, the changes in macroeconomic variables are directly incorporated to the system of equations:

$$R_{i,t+1} = C_i + A \cdot \sigma_{im,t+1} + B \cdot X_t + e_{i,t+1}, \quad (11)$$

where X_t denotes a vector of control variables; the change in the default spread ΔDEF_t , the change in the term spread $\Delta TERM_t$, and the change in the federal funds rate ΔFED_t .

Table 2 tests the significance of common slope (A) on the conditional covariance of individual stocks (portfolios) with the market portfolio after controlling for ΔDEF_t , $\Delta TERM_t$, and ΔFED_t . The first column of Table 2 shows that the risk aversion coefficient is significantly positive, in the range of 1.58 to 3.07, with the t -statistics ranging from 3.17 to 7.27. This provides evidence for a significantly positive link between expected return and risk after controlling for macroeconomic factors.

An interesting observation in Table 2 is that the common slope (B) on ΔDEF_t is positive and highly significant for all test assets, whereas the common slope on ΔFED_t is statistically insignificant. The time-series relation between returns and $\Delta TERM_t$ is negative for all portfolios and Dow 30 stocks, but the relation is not strong statistically for all test assets. The common slope on $\Delta TERM_t$ is negative and significant for the Momentum, Industry, and ROA portfolios.

The last column of Table 2 reports the Wald statistics and the p -values in square brackets from testing the joint hypothesis of all intercepts equal zero. The Wald statistics are smaller than the critical value for the size, book-to-market, industry, and IA portfolios as well as Dow 30 stocks, implying that the DCC-based conditional covariances capture the time-series and cross-sectional variation in returns on these test assets. For the Momentum and ROA portfolios, although the first essential test of the ICAPM is passed, positive common slope ($A > 0$), the model fails on the second test, zero

Table 2
Risk-return tradeoff after controlling for macroeconomic variables.

| Test assets | $\sigma_{im,t+1}$ | ΔDEF_t | $\Delta TERM_t$ | ΔFED_t | Wald |
|----------------|-------------------|----------------|-----------------|-----------------|--------------|
| Size | 1.9343 (3.17) | 1.5479 (2.49) | -0.2412 (-1.29) | 0.0043 (0.20) | 13.73 [0.19] |
| Book-to-market | 2.1126 (4.70) | 1.7678 (2.93) | -0.2516 (-1.31) | 0.0126 (0.35) | 4.64 [0.91] |
| Momentum | 3.0663 (6.97) | 2.1292 (3.61) | -0.4722 (-2.52) | 0.0263 (0.74) | 18.43 [0.05] |
| Industry | 1.5785 (3.49) | 2.0149 (3.87) | -0.4478 (-2.71) | -0.0154 (-0.49) | 8.18 [0.61] |
| IA | 1.7598 (3.69) | 1.5394 (2.50) | -0.2923 (-1.49) | 0.0052 (0.14) | 9.81 [0.46] |
| ROA | 2.3972 (5.60) | 1.7150 (2.99) | -0.3682 (-2.02) | 0.0363 (1.06) | 29.14 [0.00] |
| Dow 30 stocks | 2.1424 (7.27) | 2.1514 (3.67) | -0.2737 (-1.53) | -0.0734 (-1.62) | 18.08 [0.96] |

Entries report the common slope estimates and the t -statistics (in parentheses) from the following system of equations,

$$R_{i,t+1} = C_i + A\sigma_{im,t+1} + BX_t + e_{i,t+1},$$

where $R_{i,t+1}$ denotes the daily excess return on stock i (or portfolio i) at time $t+1$, $R_{m,t+1}$ denotes the daily excess return on the market portfolio at time $t+1$, and $\sigma_{im,t+1}$ is the time- t expected conditional covariance between $R_{i,t+1}$ and $R_{m,t+1}$. C_i is the intercept for asset i , and A and B are the common slope coefficients. X_t denotes a vector of control variables; the change in the default spread ΔDEF_t , the change in the term spread $\Delta TERM_t$, and the change in the federal funds rate ΔFED_t . The first four columns show the common slope coefficients on $\sigma_{im,t+1}$ and control variables (A, B), and the corresponding t -statistics in parentheses. The last column reports the Wald statistics and the p -values in square brackets from testing the joint hypothesis of all intercepts equal zero, $H_0: C_i = 0$.

intercepts ($H_0: C_i=0$), because the Wald statistics are large for the Momentum (p -value=0.05) and ROA (p -value=0.00) portfolios.

4.5. Controlling for unexpected news in market volatility

Earlier studies examine the significance of an intertemporal relation between the conditional mean and conditional volatility of excess returns on the market portfolio. The results from testing if the conditional volatility of the market portfolio predicts time-series variation in future returns on the market portfolio have so far been inconclusive. This section investigates whether unexpected news in future market volatility can predict time-series variation in asset returns. Tests are conducted to determine whether incorporating market volatility risk into the ICAPM has any impact on the daily risk-return tradeoff. The significance of unexpected news in market volatility is determined by estimating the following system of equations:

$$R_{i,t+1} = C_i + A \cdot \sigma_{im,t+1} + B \cdot \Delta VOL_{m,t} + e_{i,t+1}, \quad (12)$$

where $\Delta VOL_{m,t}$ is the change in the implied market volatility (VXO_t) obtained from the S&P 100 index options.

Table 3 provides strong evidence for a significant link between expected returns on individual stocks (portfolios) and their conditional covariances with the market portfolio even after controlling for unexpected news in future market volatility. For all test assets, the risk-return coefficients (A) are estimated to be positive, in the range of 1.70–3.16, and highly significant with the t -statistics ranging from 3.78 to 7.40. Another notable point in Table 3 is that the common slope (B) on the change in options' implied volatility is found to be positive and statistically significant for all portfolios and Dow 30 stocks. These results indicate that an increase in daily market volatility brings about an increase in expected returns on individual stocks and portfolios over the next trading day.

Similar to our findings in Table 2, the Wald statistics from testing the joint hypothesis of zero intercepts are insignificant for the size, book-to-market, industry, and IA portfolios and Dow 30 stocks, resurrecting the conditional CAPM for these test assets. However, the Wald statistics are still large for the Momentum and ROA portfolios. Although the risk aversion coefficient is estimated to be positive and significant for these portfolios, the tests reject the equality of zero intercepts because the time-varying conditional covariances do not capture the extremely large cross-sectional spreads between returns on winner-loser momentum and high-low ROA portfolios.

4.6. Risk-return tradeoff with intertemporal hedging demand

This section tests the significance of risk premium induced by the conditional covariation with the market portfolio after controlling for risk premia induced by the conditional covariation of asset returns with macroeconomic variables, financial factors, and volatility measures.

4.6.1. Risk premia induced by conditional covariation with macroeconomic variables

Financial economists often choose certain macroeconomic variables to control for stochastic shifts in the investment opportunity set. The widely used variables include the short-term interest rates, default spreads on corporate bond yields, and term spreads on Treasury yields. To investigate how these macroeconomic variables vary with the investment opportunity and whether covariations of individual stocks (portfolios) with them induce additional risk premia, we first

Table 3
Risk-return tradeoff after controlling for market volatility.

| Test assets | $\sigma_{im,t+1}$ | $\Delta VOL_{m,t}$ | Wald |
|----------------|-------------------|--------------------|--------------|
| Size | 2.1969 (5.95) | 0.0344 (2.36) | 12.22 [0.27] |
| Book-to-market | 2.2008 (4.92) | 0.0121 (2.16) | 4.66 [0.91] |
| Momentum | 3.1630 (7.20) | 0.0307 (5.60) | 18.54 [0.05] |
| Industry | 1.7042 (3.78) | 0.0247 (5.09) | 8.08 [0.62] |
| IA | 1.8406 (3.87) | 0.0290 (5.06) | 9.76 [0.46] |
| ROA | 2.4507 (5.73) | 0.0310 (5.81) | 29.11 [0.00] |
| Dow 30 stocks | 2.1800 (7.40) | 0.0181 (3.53) | 18.15 [0.96] |

Entries report the common slope estimates and the t -statistics (in parentheses) from the following system of equations:

$$R_{i,t+1} = C_i + A \cdot \sigma_{im,t+1} + B \cdot \Delta VOL_{m,t} + e_{i,t+1},$$

where $R_{i,t+1}$ denotes the daily excess return on stock i (or portfolio i) at time $t+1$, $R_{m,t+1}$ denotes the daily excess return on the market portfolio at time $t+1$, $\sigma_{im,t+1}$ is the time- t expected conditional covariance between $R_{i,t+1}$ and $R_{m,t+1}$, and $\Delta VOL_{m,t}$ is the change in the implied market volatility (VXO_t) obtained from the S&P 100 index options. C_i is the intercept for asset i , and A and B are the common slope coefficients. The first column shows the common slope coefficient (A) on $\sigma_{im,t+1}$, the second column presents the common slope coefficient (B) on $\Delta VOL_{m,t}$. The t -statistics in are reported in parentheses. The last column reports the Wald statistics and the p -values in square brackets from testing the joint hypothesis of all intercepts equal zero, $H_0: C_i=0$.

Table 4
Risk premia induced by conditional covariation with macroeconomic variables.

| Test assets | $\sigma_{im,t+1}$ | $\sigma_{i,\Delta DEF,t+1}$ | $\sigma_{i,\Delta TERM,t+1}$ | $\sigma_{i,\Delta FED,t+1}$ | Wald |
|---------------|-------------------|-----------------------------|------------------------------|-----------------------------|--------------|
| Momentum | 3.7563 (6.20) | 12.527 (2.48) | -2.2856 (-2.45) | -0.4076 (-1.82) | 26.25 [0.00] |
| ROA | 4.5288 (6.86) | 17.415 (3.45) | -2.1571 (-1.98) | 0.2189 (1.25) | 26.12 [0.00] |
| Dow 30 stocks | 2.5524 (5.87) | 11.884 (3.58) | -2.5583 (-3.67) | -0.1403 (-0.32) | 21.10 [0.88] |

Entries report the common slope estimates and the t -statistics (in parentheses) from the following system of equations,

$$R_{i,t+1} = C_i + A \cdot \sigma_{im,t+1} + B_1 \cdot \sigma_{i,\Delta DEF,t+1} + B_2 \cdot \sigma_{i,\Delta TERM,t+1} + B_3 \cdot \sigma_{i,\Delta FED,t+1} + e_{i,t+1},$$

where $\sigma_{im,t+1}$ measures the time- t expected conditional covariance between the excess returns on stock i (or portfolio i) and the market portfolio, $\sigma_{i,\Delta DEF,t+1}$ measures the time- t expected conditional covariance between $R_{i,t+1}$ and the change in the default spread (ΔDEF_{t+1}), $\sigma_{i,\Delta TERM,t+1}$ is the time- t expected conditional covariance between $R_{i,t+1}$ and the change in the term spread ($\Delta TERM_{t+1}$), and $\sigma_{i,\Delta FED,t+1}$ is the time- t expected conditional covariance between $R_{i,t+1}$ and the change in the federal funds rate (ΔFED_{t+1}). C_i is the intercept for asset i , and A , B_1 , B_2 , and B_3 are the common slope coefficients. The t -statistics are given in parentheses. The last column reports the Wald statistics and the p -values in square brackets from testing the joint hypothesis of all intercepts equal zero, $H_0: C_i = 0$.

estimate the conditional covariance of these variables with excess returns on each stock (portfolio) and then analyze how the stocks' (portfolio's) excess returns respond to their conditional covariance with these economic factors. In estimating the conditional covariances, the changes in default spreads, term spreads, and federal funds rates are used.

Table 4 reports the common slope estimates (A , B_1 , B_2 , B_3) and their t -statistics from the following system of equations:

$$R_{i,t+1} = C_i + A \cdot \sigma_{im,t+1} + B_1 \cdot \sigma_{i,\Delta DEF,t+1} + B_2 \cdot \sigma_{i,\Delta TERM,t+1} + B_3 \cdot \sigma_{i,\Delta FED,t+1} + e_{i,t+1}, \quad (13)$$

where $\sigma_{i,\Delta DEF,t+1}$, $\sigma_{i,\Delta TERM,t+1}$, and $\sigma_{i,\Delta FED,t+1}$ measure the time- t expected conditional covariance between $R_{i,t+1}$ and the change in the default spread (ΔDEF_{t+1}), the change in the term spread ($\Delta TERM_{t+1}$), and the change in the federal funds rate (ΔFED_{t+1}), respectively.

The parameter estimates in Table 4 reveal several important results.⁷ First, incorporating the covariance of asset returns with any of these macroeconomic variables does not alter the magnitude and statistical significance of the risk aversion estimates. In all cases, the common slope coefficient (A) on $\sigma_{im,t+1}$ is positive, in the range of 2.55 and 4.53, and highly significant with the t -statistics between 5.87 and 6.86. Second, the slope coefficient (B_1) on $\sigma_{i,\Delta DEF,t+1}$ is positive and significant with the t -statistics ranging from 2.48 to 3.58. The positive slope on $\sigma_{i,\Delta DEF,t+1}$ indicates that individual stocks and equity portfolios that have higher covariance with the change in default spreads are expected to generate higher returns next period. Third, the common slope (B_2) on $\sigma_{i,\Delta TERM,t+1}$ is negative and significant even though the relation is somewhat weaker for the ROA portfolios. The negative slope on $\sigma_{i,\Delta TERM,t+1}$ implies that stocks (portfolios) that have higher association with the change in term spread are expected to generate lower returns next period. Among the macroeconomic variables, the federal funds rate does not have significant risk premia since the conditional covariance of returns with the change in fed funds rate does not predict the time-series variation in stock and portfolio returns.

4.6.2. Risk premia induced by conditional covariation with financial factors

This section takes the size (*SMB*), book-to-market (*HML*), and momentum (*MOM*) factors of Fama-French (1993) and Carhart (1997), and the investment-to-assets (*IA*) and the return-on-assets (*ROA*) factors of Chen and Zhang (2009) to describe the state of the investment opportunity and to examine whether covariations of stocks (portfolios) with these financial factors induce additional risk premia. The conditional covariances of each stock (portfolio) with the financial factors are estimated first and then the following SUR panel regression is run:

$$R_{i,t+1} = C_i + A \cdot \sigma_{im,t+1} + B_1 \cdot \sigma_{i,SMB,t+1} + B_2 \cdot \sigma_{i,HML,t+1} + B_3 \cdot \sigma_{i,MOM,t+1} + e_{i,t+1}, \quad (14)$$

$$R_{i,t+1} = C_i + A \cdot \sigma_{im,t+1} + B_4 \cdot \sigma_{i,IA,t+1} + B_5 \cdot \sigma_{i,ROA,t+1} + e_{i,t+1},$$

where $\sigma_{i,SMB,t+1}$ is the time- t conditional covariance between $R_{i,t+1}$ and SMB_{t+1} , $\sigma_{i,HML,t+1}$ is the time- t conditional covariance between $R_{i,t+1}$ and HML_{t+1} , $\sigma_{i,MOM,t+1}$ is the time- t conditional covariance between $R_{i,t+1}$ and MOM_{t+1} , $\sigma_{i,IA,t+1}$ is the time- t conditional covariance between $R_{i,t+1}$ and IA_{t+1} , and $\sigma_{i,ROA,t+1}$ is the time- t conditional covariance between $R_{i,t+1}$ and ROA_{t+1} . From the estimates of B_1 , B_2 , B_3 , B_4 , and B_5 , one can learn how investors react to the covariations of stock returns with financial factors.

Table 5 provides strong evidence for a significant link between expected returns on stocks (portfolios) and their conditional covariances with the market after controlling for risk premia induced by the conditional covariation with the *SMB*, *HML*, *MOM*, *IA*, and *ROA* factors. The risk-return coefficients (A) are estimated to be in the range of 2.72 to 4.52 and highly significant with the t -statistics ranging from 2.42 to 8.67. The conditional covariances of stock (portfolio) returns with the size and momentum factors do not have robust, significant predictive power for one day ahead returns on Dow 30

⁷ Starting with Table 4 results are presented for the Momentum and ROA portfolios, and Dow 30 stocks to save space.

Table 5
Risk premia induced by conditional covariation with financial factors.

| Test assets | $\sigma_{im,t+1}$ | $\sigma_{i,SMB,t+1}$ | $\sigma_{i,HML,t+1}$ | $\sigma_{i,MOM,t+1}$ | $\sigma_{i,IA,t+1}$ | $\sigma_{i,ROA,t+1}$ | Wald |
|---------------|-------------------|----------------------|----------------------|----------------------|---------------------|----------------------|--------------|
| Momentum | 2.3999 (2.42) | 0.3258 (0.10) | 0.6487 (0.17) | -4.3437 (-1.71) | | | 32.21 [0.00] |
| Momentum | 4.5230 (6.20) | | | | 16.091 (2.22) | 6.6121 (2.61) | 29.83 [0.00] |
| ROA | 2.9451 (3.36) | -3.3770 (-1.22) | 5.9732 (2.38) | 1.1687 (0.64) | | | 15.72 [0.11] |
| ROA | 4.1639 (6.62) | | | | 11.982 (2.11) | 3.5994 (3.37) | 12.86 [0.23] |
| Dow 30 stocks | 2.7215 (5.75) | -3.8006 (-1.27) | 8.5071 (2.72) | 3.9670 (2.01) | | | 16.84 [0.97] |
| Dow 30 stocks | 3.5959 (8.67) | | | | 17.886 (2.49) | 2.2097 (0.68) | 15.62 [0.99] |

Entries report the common slope estimates and the t -statistics (in parentheses) from the following system of equations:

$$R_{i,t+1} = C_i + A \cdot \sigma_{im,t+1} + B_1 \cdot \sigma_{i,SMB,t+1} + B_2 \cdot \sigma_{i,HML,t+1} + B_3 \cdot \sigma_{i,MOM,t+1} + e_{i,t+1},$$

$$R_{i,t+1} = C_i + A \cdot \sigma_{im,t+1} + B_4 \cdot \sigma_{i,IA,t+1} + B_5 \cdot \sigma_{i,ROA,t+1} + e_{i,t+1},$$

where $\sigma_{im,t+1}$ is the time- t expected conditional covariance between the excess returns on stock i (or portfolio i) and the market portfolio, $\sigma_{i,SMB,t+1}$ is the time- t conditional covariance between $R_{i,t+1}$ and SMB_{t+1} , $\sigma_{i,HML,t+1}$ is the time- t conditional covariance between $R_{i,t+1}$ and HML_{t+1} , $\sigma_{i,MOM,t+1}$ is the time- t conditional covariance between $R_{i,t+1}$ and MOM_{t+1} , $\sigma_{i,IA,t+1}$ is the time- t conditional covariance between $R_{i,t+1}$ and IA_{t+1} , and $\sigma_{i,ROA,t+1}$ is the time- t conditional covariance between $R_{i,t+1}$ and ROA_{t+1} . C_i is the intercept for asset i , and A , B_1 , B_2 , B_3 , B_4 , and B_5 are the common slope coefficients. The t -statistics are reported in parentheses. The last column reports the Wald statistics and the p -values in square brackets from testing the joint hypothesis of all intercepts equal zero, $H_0: C_i = 0$.

stocks, Momentum, and ROA portfolios. In other words, the *SMB* and *MOM* factors are not consistently priced in the ICAPM framework.

Another notable point in Table 5 is that the common slopes (B_2 , B_4 , B_5) on $\sigma_{i,HML,t+1}$, $\sigma_{i,IA,t+1}$, and $\sigma_{i,ROA,t+1}$ are positive in all cases and statistically significant for most test assets considered in the paper. Thus, an increase in the covariance of a stock (portfolio) return with the *HML*, *IA*, and *ROA* factors predicts an increase in the stock's (portfolio's) expected excess return over the next trading day. The positive slope estimates on $\sigma_{i,HML,t+1}$, $\sigma_{i,IA,t+1}$, and $\sigma_{i,ROA,t+1}$ suggest that upward movements in the *HML*, *IA*, and *ROA* factors predict favorable shifts in the investment opportunity set, implying that these are priced factors that are correlated with innovations in investment opportunities.⁸

4.6.3. Risk premium induced by conditional covariation with market volatility risk

Following Campbell (1993, 1996), investors are assumed to hedge against unexpected changes in future market volatility defined here as the first-difference of the options implied volatility of S&P 100 index return ($\Delta VXO_{m,t+1}$). This section tests whether stocks (portfolios) that have higher correlation with the change in future market volatility yield lower expected return.

When considering stochastic investment opportunities governed by innovations in future market volatility, the intertemporal relation is estimated from the following system of equations:

$$R_{i,t+1} = C_i + A \cdot \sigma_{im,t+1} + B \cdot \sigma_{i,\Delta VOL_{m,t+1}} + e_{i,t+1}, \quad (15)$$

where $\sigma_{i,\Delta VOL_{m,t+1}}$ measures the time- t expected conditional covariance between $R_{i,t+1}$ and the change in expected future volatility of the market portfolio denoted by $\Delta VOL_{m,t+1}$.⁹

Under the null hypothesis of Campbell's (1993, 1996) ICAPM, the common slope (A) on $\sigma_{im,t+1}$ should be positive and significant, and the common slope (B) on $\sigma_{i,\Delta VOL_{m,t+1}}$ should be negative and significant. As shown in Table 6, the risk-return coefficient (A) on $\sigma_{im,t+1}$ is estimated to be in the range of 1.57 to 2.07 with the t -statistics ranging from 2.76 to 4.17, implying a positive intertemporal relation between expected return and risk.

The common slope (B) on $\sigma_{i,\Delta VOL_{m,t+1}}$ is estimated to be negative, between -0.31 and -0.76, and highly significant with the t -statistics ranging from -2.60 to -4.30. These results imply a negative intertemporal relation between expected return and volatility risk.¹⁰ In other words, stocks (portfolios) that have higher correlation with the changes in future market volatility yield lower expected return next period.

⁸ Our finding that the *HML* is a priced factor is consistent with the recent empirical evidence provided by Campbell and Vuolteenaho (2004), Brennan et al. (2004), and Petkova and Zhang (2005) as well as with the recent theoretical models of Gomes et al. (2003) and Zhang (2005). However, the explanation of value premium within the conditional CAPM framework is still a subject of an intense debate. Lettau and Ludvigson (2001) and Ang and Chen (2007) find that the conditional CAPM helps explain the value premium, but Lewellen and Nagel (2006) provide evidence that is not in agreement with the findings of Ang and Chen (2007).

⁹ At an earlier stage of the study, three alternative measures of $\Delta VOL_{m,t+1}$ are used: (1) The change in the GARCH conditional volatility of S&P 500 index; (2) The change in the option implied volatility of S&P 100 index; and (3) The change in the range volatility of S&P 500 index. The qualitative results turn out to be similar and they are available in the online supplement. To save space, results are reported only for the options' implied volatility.

¹⁰ Bakshi and Kapadia (2003) find the volatility risk premium to be negative in index options markets. We examine whether the volatility risk premium is negative within the ICAPM framework of Campbell (1993, 1996) using individual stocks and equity portfolios.

Table 6
Risk premium induced by conditional covariation with unexpected market volatility.

| Test assets | $\sigma_{im,t+1}$ | $\sigma_{i,\Delta VOL_{m,t+1}}$ | Wald |
|---------------|-------------------|---------------------------------|--------------|
| Momentum | 1.6810 (2.76) | -0.7550 (-4.30) | 17.21 [0.07] |
| ROA | 2.0736 (4.17) | -0.5183 (-2.79) | 26.57 [0.00] |
| Dow 30 stocks | 1.5712 (4.02) | -0.3073 (-2.60) | 17.29 [0.98] |

Entries report the common slope estimates and the t -statistics (in parentheses) from the following system of equations:

$$R_{i,t+1} = C_i + A \cdot \sigma_{im,t+1} + B \cdot \sigma_{i,\Delta VOL_{m,t+1}} + \epsilon_{i,t+1},$$

where $\sigma_{im,t+1}$ measures the time- t expected conditional covariance between the excess returns on stock i (or portfolio i) and the market portfolio. $\sigma_{i,\Delta VOL_{m,t+1}}$ measures the time- t expected conditional covariance between $R_{i,t+1}$ and the change in the option implied volatility of S&P 100 index ($\Delta VXO_{m,t+1}$). C_i is the intercept for asset i , and A and B are the common slope coefficients. The first column shows the common slope coefficient (A) on $\sigma_{im,t+1}$, and the second column presents the common slope coefficient (B) on $\sigma_{i,\Delta VOL_{m,t+1}}$. The t -statistics in are reported in parentheses. The last column reports the Wald statistics and the p -values in square brackets from testing the joint hypothesis of all intercepts equal zero, $H_0: C_i = 0$.

These results are consistent with Campbell (1993, 1996) that provides a two-factor ICAPM in which an unexpected increase in future market volatility represents deterioration in the investment opportunity set or a decrease in optimal consumption. As shown in Table 3, there is a positive and significant link between returns and innovations in market volatility. Risk-averse investors will demand more of an asset, the more positively correlated the asset's return is with changes in market volatility because they will be compensated by a higher level of wealth through positive correlation of the returns. That asset can be viewed as a hedging instrument. In other words, an increase in the covariance of returns with volatility risk leads to an increase in the hedging demand, which in equilibrium reduces expected return on the asset. Thus, in the context of Campbell's (1993, 1996) ICAPM, a positive covariance of returns with volatility shocks (or innovations in market volatility) predicts a lower return on the risky asset.

5. Conclusion

The daily intertemporal relation is estimated between expected return and risk using a cross section of 30 stocks in the Dow Jones Industrial Average as well as various equity portfolios. By so doing, not only the cross-sectional consistency of the intertemporal relation is guaranteed, but also additional statistical power is gained by pooling multiple time-series and cross-section together for a joint estimation with common slope coefficients. The average relative risk aversion is estimated to be positive, highly significant, and robust to variations in the market portfolios, sample periods, data sets, estimation methodologies, and the conditional mean and covariance specifications. The positive risk-return tradeoff remains intact after controlling for unexpected news in macroeconomic variables and market volatility. The magnitude of the risk-return coefficient is also economically sensible, ranging from two to four.

When investigating the intertemporal hedging demands and the associated risk premia induced by the conditional covariation of test assets with a set of macroeconomic variables, the common slopes on the conditional covariances of returns with the changes in default and term spreads are found to be statistically significant, implying that the innovations in these macro factors contain systematic risks rewarded in the stock market. Statistical tests are conducted to determine whether the *SMB*, *HML*, *MOM*, *IA*, and *ROA* factors move closely with investment opportunities and whether covariations with these financial factors induce additional risk premia on individual stocks and portfolios. The results indicate that although the *SMB* and *MOM* factors are not priced in the ICAPM framework, the *HML*, *IA*, and *ROA* are priced factors and can be viewed as a proxy for investment opportunities. Finally, investors are assumed to hedge against the changes in future market volatility and options' implied volatility is used to test if assets that have higher correlation with the innovations in future market volatility yield lower expected return. The parameter estimates provide strong evidence for a significantly negative relation between expected return and volatility risk. However, incorporating the conditional covariation with any of these state variables does not change the positive risk premium induced by the conditional covariation of stock returns with the market portfolio.

Appendix A. Supplementary material

Supplementary data associated with this article can be found in the online version at [doi:10.1016/j.jmoneco.2010.03.002](https://doi.org/10.1016/j.jmoneco.2010.03.002).

References

- Abel, A.B., 1988. Stock prices under time-varying dividend risk: an exact solution in an infinite-horizon general equilibrium model. *Journal of Monetary Economics* 22, 375–393.

- Alizadeh, S., Brandt, M.W., Diebold, F.X., 2002. Range-based estimation of stochastic volatility models. *Journal of Finance* 57, 1047–1092.
- Ang, A., Chen, J., 2007. CAPM over the long-run: 1926–2001. *Journal of Empirical Finance* 14, 1–40.
- Backus, D.K., Gregory, A.W., 1993. Theoretical relations between risk premiums and conditional variance. *Journal of Business and Economic Statistics* 11, 177–185.
- Baillie, R.T., DeGennaro, R.P., 1990. Stock returns and volatility. *Journal of Financial and Quantitative Analysis* 25, 203–214.
- Bakshi, G., Kapadia, N., 2003. Delta hedged gains and the negative market volatility risk premium. *Review of Financial Studies* 16, 527–566.
- Bali, T.G., 2008. The intertemporal relation between expected returns and risk. *Journal of Financial Economics* 87, 101–131.
- Bali, T.G., Engle, R.F., 2007. Investigating ICAPM with dynamic conditional correlations. Working Paper, Stern School of Business, New York University.
- Black, F., Scholes, M., 1973. The pricing of options and corporate liabilities. *Journal of Political Economy* 81, 637–654.
- Bollerslev, T., 1990. Modeling the coherence in short-run nominal exchange rates: a multivariate generalized ARCH model. *Review of Economics and Statistics* 72, 498–505.
- Bollerslev, T., Engle, R.F., Wooldridge, J.M., 1988. A capital asset pricing model with time-varying covariances. *Journal of Political Economy* 96, 116–131.
- Bollerslev, T., Wooldridge, J.M., 1992. Quasi-maximum likelihood estimation and inference in dynamic models with time varying covariances. *Econometric Reviews* 11, 143–172.
- Bollerslev, T., Zhou, H., 2006. Volatility puzzles: a simple framework for gauging return-volatility regressions. *Journal of Econometrics* 131, 123–150.
- Brandt, M.W., Diebold, F.X., 2006. A no-arbitrage approach to range-based estimation of return covariances and correlations. *Journal of Business* 79, 61–73.
- Brandt, M.W., Kang, Q., 2004. On the relationship between the conditional mean and volatility of stock returns: a latent VAR approach. *Journal of Financial Economics* 72, 217–257.
- Brennan, M., Wang, A., Xia, Y., 2004. Estimation and test of a simple model of intertemporal asset pricing. *Journal of Finance* 59, 1743–1775.
- Breen, W., Glosten, L.R., Jagannathan, R., 1989. Economic significance of predictable variations in stock index returns. *Journal of Finance* 44, 1177–1189.
- Campbell, J.Y., 1987. Stock returns and the term structure. *Journal of Financial Economics* 18, 373–399.
- Campbell, J.Y., 1993. Intertemporal asset pricing without consumption data. *American Economic Review* 83, 487–512.
- Campbell, J.Y., 1996. Understanding risk and return. *Journal of Political Economy* 104, 298–345.
- Campbell, J.Y., Hentchel, L., 1992. No news is good news: an asymmetric model of changing volatility in stock returns. *Journal of Financial Economics* 31, 281–318.
- Campbell, J.Y., Vuolteenaho, T., 2004. Bad beta, good beta. *American Economic Review* 94, 1249–1275.
- Carhart, M.M., 1997. On persistence in mutual fund performance. *Journal of Finance* 52, 57–82.
- Chen, L., Zhang, L., 2009. A better three-factor model that explains more anomalies. *Journal of Finance*, forthcoming.
- Ding, Z., Engle, R.F., 2001. Large-scale conditional covariance matrix modeling, estimation, and testing. *Academia Economic Papers* 2, 157–184.
- Engle, R.F., 2002. Dynamic conditional correlation: a simple class of multivariate generalized autoregressive conditional heteroskedasticity models. *Journal of Business and Economic Statistics* 20, 339–350.
- Engle, R.F., 2009a. In: *Anticipating Correlations. A New Paradigm for Risk Management*. Princeton University Press.
- Engle, R.F., 2009b. High dimensional dynamic correlations. In: Castle, J.L., Shephard, N. (Eds.), *The Methodology and Practice of Econometrics: Papers in Honour of David F. Hendry*. Oxford University Press.
- Engle, R.F., Kelly, B., 2007. Dynamic equicorrelation. Working Paper, Stern School of Business, New York University.
- Engle, R.F., Ng, V., Rothschild, M., 1990. Asset pricing with a factor ARCH covariance structure: empirical estimates for Treasury bills. *Journal of Econometrics* 45, 213–237.
- Engle, R.F., Shephard, N., Shephard, K., 2008. Fitting and testing vast dimensional time-varying covariance models. Working Paper, New York University and University of Oxford.
- Fama, E.F., French, K.R., 1992. Cross-section of expected stock returns. *Journal of Finance* 47, 427–465.
- Fama, E.F., French, K.R., 1993. Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics* 33, 3–56.
- French, K.R., Schwert, G.W., Stambaugh, R., 1987. Expected stock returns and volatility. *Journal of Financial Economics* 19, 3–29.
- Gennotte, G., Marsh, T.A., 1993. Variations in economic uncertainty and risk premiums on capital assets. *European Economic Review* 37, 1021–1041.
- Ghysels, E., Santa-Clara, P., Valkanov, R., 2005. There is a risk-return trade-off after all. *Journal of Financial Economics* 76, 509–548.
- Glosten, L.R., Jagannathan, R., Runkle, D.E., 1993. On the relation between the expected value and the volatility of the nominal excess returns on stocks. *Journal of Finance* 48, 1779–1801.
- Gomes, J., Kogan, L., Zhang, L., 2003. Equilibrium cross section of returns. *Journal of Political Economy* 111, 693–732.
- Guo, H., Whitelaw, R.F., 2006. Uncovering the risk-return relation in the stock market. *Journal of Finance* 61, 1433–1463.
- Harrison, P., Zhang, H., 1999. An investigation of the risk and return relation at long horizons. *Review of Economics and Statistics* 81, 399–408.
- Harvey, C.R., 2001. The specification of conditional expectations. *Journal of Empirical Finance* 8, 573–637.
- Jegadeesh, N., Titman, S., 1993. Returns to buying winners and selling losers: implications for stock market efficiency. *Journal of Finance* 48, 65–91.
- Lettau, M., Ludvigson, S., 2001. Resurrecting the (C)CAPM: a cross-sectional test when risk premia are time-varying. *Journal of Political Economy* 109, 1238–1287.
- Lewellen, J., Nagel, S., 2006. The conditional CAPM does not explain asset-pricing anomalies. *Journal of Financial Economics* 82, 289–314.
- Lundblad, C., 2007. The risk return tradeoff in the long run: 1836–2003. *Journal of Financial Economics* 85, 123–150.
- Merton, R.C., 1973. An intertemporal asset pricing model. *Econometrica* 41, 867–887.
- Merton, R.C., 1980. On estimating the expected return on the market: an exploratory investigation. *Journal of Financial Economics* 8, 323–361.
- Nelson, D.B., 1991. Conditional heteroskedasticity in asset returns: a new approach. *Econometrica* 59, 347–370.
- Petkova, R., Zhang, L., 2005. Is value riskier than growth? *Journal of Financial Economics* 78, 187–202.
- Ross, S.A., 1976. The arbitrage theory of capital asset pricing. *Journal of Economic Theory* 13, 341–360.
- Scruggs, J.T., 1998. Resolving the puzzling intertemporal relation between the market risk premium and conditional market variance: a two-factor approach. *Journal of Finance* 53, 575–603.
- Tse, Y., 2000. A test for constant correlations in a multivariate GARCH model. *Journal of Econometrics* 98, 107–127.
- Whitelaw, R.F., 1994. Time variations and covariations in the expectation and volatility of stock market returns. *Journal of Finance* 49, 515–541.
- Zhang, L., 2005. The value premium. *Journal of Finance* 60, 67–103.