Basic Scheduling Algorithms for Single Machine Problems: Processing Jobs With Preemption

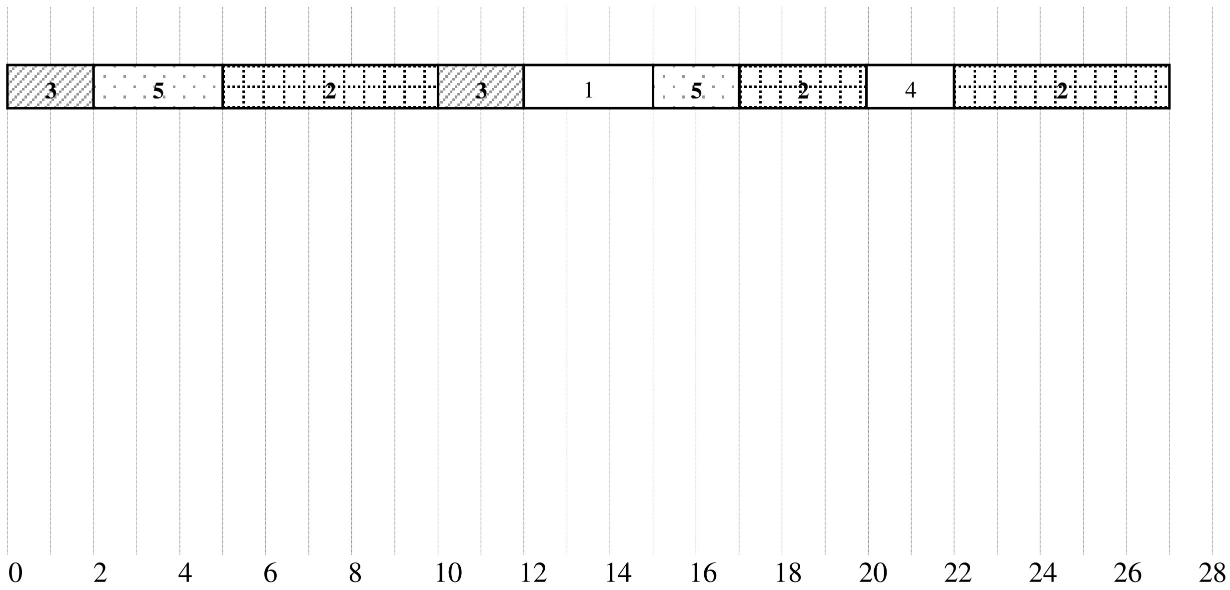
Consider a problem of scheduling jobs with preemption. In this problem, processing of any job can be interrupted and resumed later, the total processing time of all parts of job j being equal to p_j.

The following exercise demonstrates that **if all jobs are released at the same time (r_j=0)**, there is no advantage to process the jobs with preemption. In other words, for problem $1|pmtn|\Sigma C_j$ there exists an optimal schedule without preemption.

An instance of problem $1|pmtn|\Sigma C_j$ with n=5 jobs is given by job processing times
$p_1 = 3$; $p_2 = 13$; $p_3 = 4$; $p_4 = 2$; $p_5 = 5$.

Schedule S is represented by the Gantt chart below.

Construct schedule S' without preemption and without increasing completion times of all jobs.



Consider the general problem $1|pmtn|\Sigma C_j$ with n jobs and arbitrary processing times. To show that there exists an optimal nonpreemptive schedule describe a transformation that modifies an arbitrary preemptive schedule S into a nonpreemptive schedule S' without increasing the completion times of all jobs.

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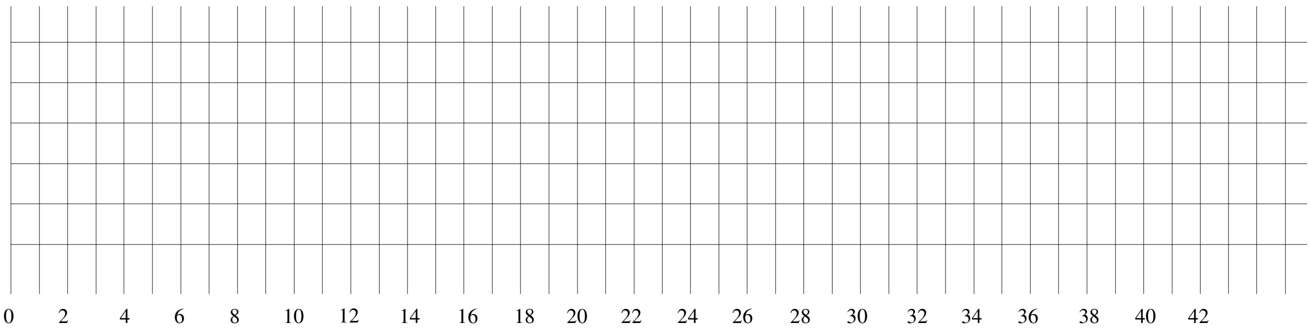
Minimising total completion time with nonzero release dates and preemption allowed: $1|r_j, pmtn|\Sigma C_j$

Consider $1|r_j, pmtn|\Sigma C_j$.

Shortest Remaining Processing Time (SRPT) rule: each time that a job is completed, or at the next release date, the job to be processed next has the smallest remaining processing time among the available jobs.

Example:

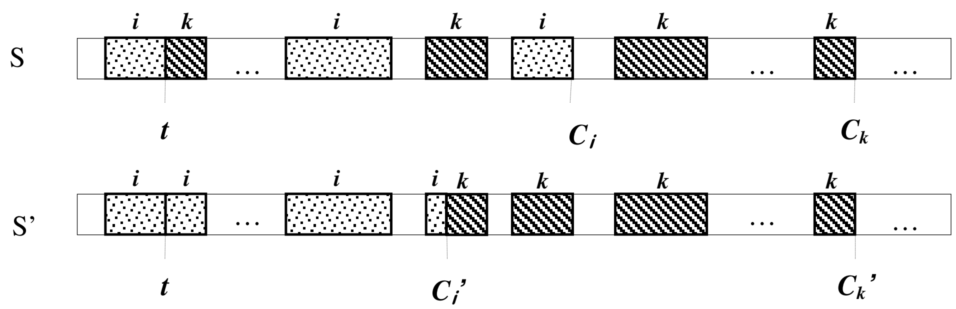
Job	r_j	p_j
1	0	13
2	3	6
3	14	8
4	18	3
5	22	5
6	25	3
7	30	4
8	33	1



Theorem 2. For problem $1|r_j, Pmtn|\Sigma C_j$, the SRPT rule is optimal.

Proof (pairwise interchange argument)

Consider a schedule in which available job i with the shortest remaining processing time is not being processed at time t, and instead available job k is being processed. Let p_i' and p_k' denote the remaining processing times for jobs i and k after time t, so $p_i' < p_k'$.
In total $p_i' + p_k'$ is spent on jobs i and k after time t. We assume that $C_i < C_k$.



Interchange:

1) Take the first p_i' units of time that were devoted to either of jobs i and k after time t, and use them instead to process job i to completion.

2) Take the remaining p_k' units of time that were spent processing i and k after time t, and use them to schedule job k.

We have obtained a 'better' schedule S':

$$C_i' < C_i$$

$$C_k' = C_k$$

Since in the new schedule all jobs other than i and k have the same the completion times as before, we obtain:

$$\sum_{j=1}^{n} C_j' - \sum_{j=1}^{n} C_j = \left(C_i' + C_k'\right) - \left(C_i + C_k\right) < 0.$$

This contradicts the optimality of schedule S. ∎