



**UNIVERSITY AT BUFFALO**

*State University of New York*

***THE SCHOOL OF ENGINEERING  
AND APPLIED SCIENCES***

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# **IE 661**

## **Scheduling Theory**

### **Chapter 2**

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# Chapter 2: Deterministic Models Preliminaries

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- Processing time  $p_{ij}$
- Release date  $r_j$
- Due date  $d_j$
- Weight  $w_j$
- Notation
  - $\alpha \mid \beta \mid \gamma$
  - $\alpha$  Machine environment
  - $\beta$  Processing characteristics and constraints
  - $\gamma$  Objective



# Chapter 2: Deterministic Models Preliminaries

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- $\alpha$  Machine environment
  - Single machine (1)
  - Identical machines in parallel ( $P_m$ )
  - Machines in parallel with different speeds ( $Q_m$ )
  - Unrelated machines in parallel ( $R_m$ )
  - Flow shop ( $F_m$ ) ( $m$  machines in series)
  - Flexible flow shop ( $FF_c$ ) ( $c$  stages with poss. Identical machines)
  - Job shop ( $J_m$ ) (*recrc* for recirculation in  $\beta$  field)
  - Flexible job shop ( $FJ_c$ )
  - Open shop ( $O_m$ ) (scheduler can determine route)



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- $\beta$  Processing characteristics and constraints
  - Release dates ( $r_j$ )
  - Sequence dependent setup times ( $s_{jk}$ )
  - Machine specific sequence dependent setup times ( $s_{ijk}$ )
  - Preemptions ( $prmp$ )
  - Precedence constraints ( $prec$ )
  - Breakdowns ( $brkdwn$ )
  - Machine eligibility restrictions ( $M_j$ )
  - Permutation ( $prmu$ )
  - Blocking ( $block$ )
  - No-wait ( $nwt$ )
  - Recirculation ( $recrc$ )



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- $\gamma$  Objective
  - Makespan ( $C_{\max}$ )
  - Max lateness ( $L_{\max}$ );  $L_j = C_j - d_j$
  - Total weighted completion time (  $\sum w_j C_j$  )
  - Discounted total weighted completion time (  $\sum w_j (1 - e^{-rC_j})$  )
  - Total weighted tardiness (  $\sum w_j T_j$  )
  - Weighted number of tardy jobs (  $\sum w_j U_j$  )
- Examples
- $Fm \mid p_{ij} = p_j \mid \sum w_j C_j$



## Chapter 2: Deterministic Models Preliminaries

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- Classes of Schedules
  - **Nondelay Schedule:** A feasible schedule is called non-delay if no machine is kept idle while an operation is waiting for processing (i.e., it prohibits *unforced idleness*).
  - A scheduling anomaly: Consider a  $P2 \mid prec \mid C_{\max}$  with the following processing times

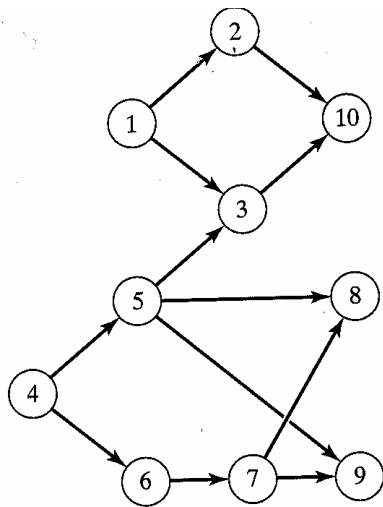
j	1	2	3	4	5	6	7	8	9	1
p <sub>j</sub>	8	7	7	2	3	2	2	8	8	0

5

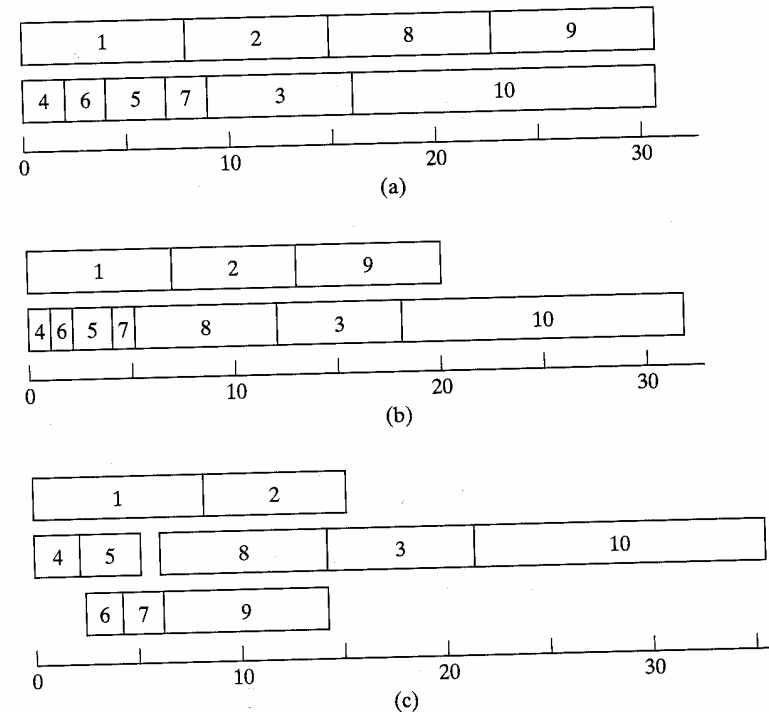


# Chapter 2: Deterministic Models

## Preliminaries



**Figure 2.2** Precedence constraints graph for Example 2.3.2.



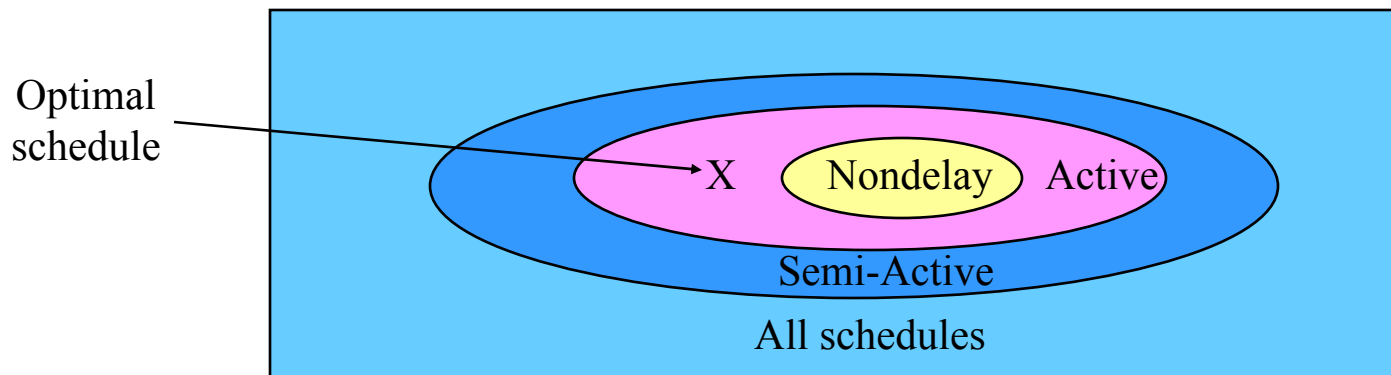
**Figure 2.3** Gantt charts of nondelay schedules: (a) original schedules, (b) processing times one unit less, and (c) original processing times and three machines.



# Chapter 2: Deterministic Models Preliminaries

- **Classes of Schedules**

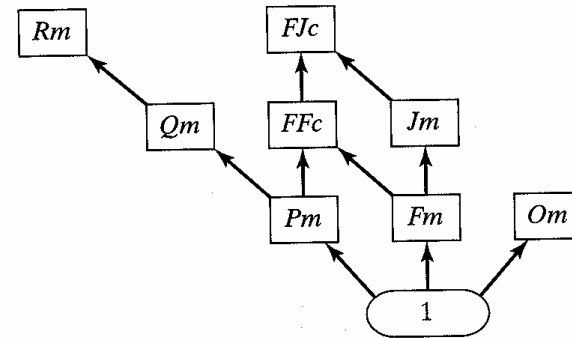
- **Active Schedule:** A feasible schedule is called active if it is not possible to construct another schedule by changing the order of processing on the machines and having at least one operation finishing earlier and no operation finishing later.
- **Semi-active Schedule:** A feasible schedule is called semi-active if no operation can be completed earlier without changing the order of processing on any one of the machines.



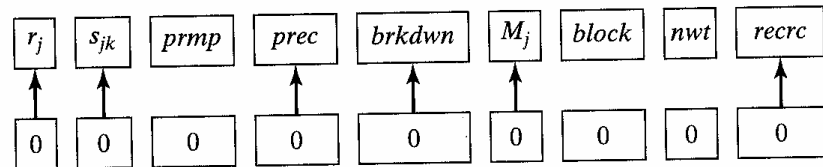


## Complexity Hierarchy

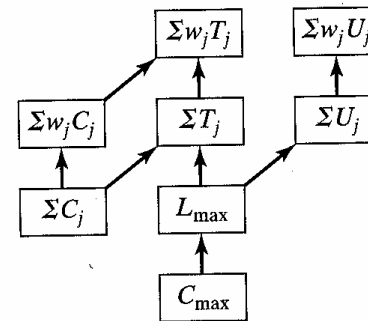
- $1 \parallel \sum C_j \alpha$   
(reduces to)  
 $1 \parallel \sum w_j C_j$
- $1 \parallel \sum w_j C_j \alpha$   
 $P_m \parallel \sum w_j C_j \alpha$   
 $Q_m \mid prec \mid \sum w_j$   
 $C_j$



(a)



(b)



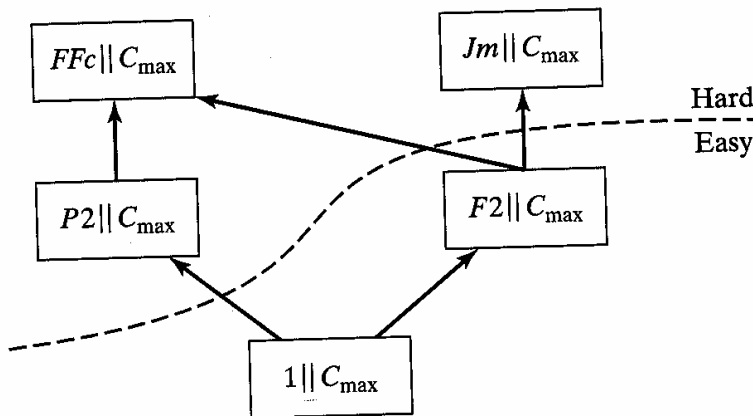
(c)

**Figure 2.7** Complexity hierarchies of deterministic scheduling problems: (1) machine environments, (b) processing restrictions and constraints, and (c) objective functions.

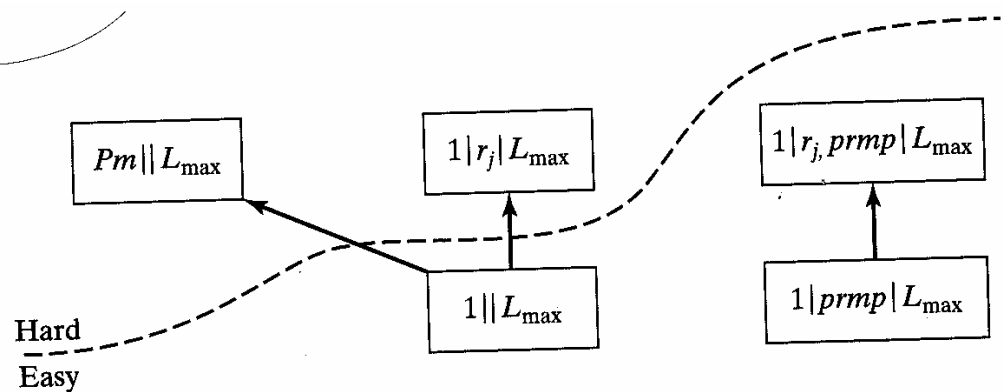


# Chapter 2: Deterministic Models Preliminaries

- Complexity hierarchy for Makespan (Fig. 2.8) and Maximum Lateness problems (Fig. 2.9)



**Figure 2.8** Complexity hierarchy of problems in Example



**Figure 2.9** Complexity hierarchy of problems in Example 2.4.2.