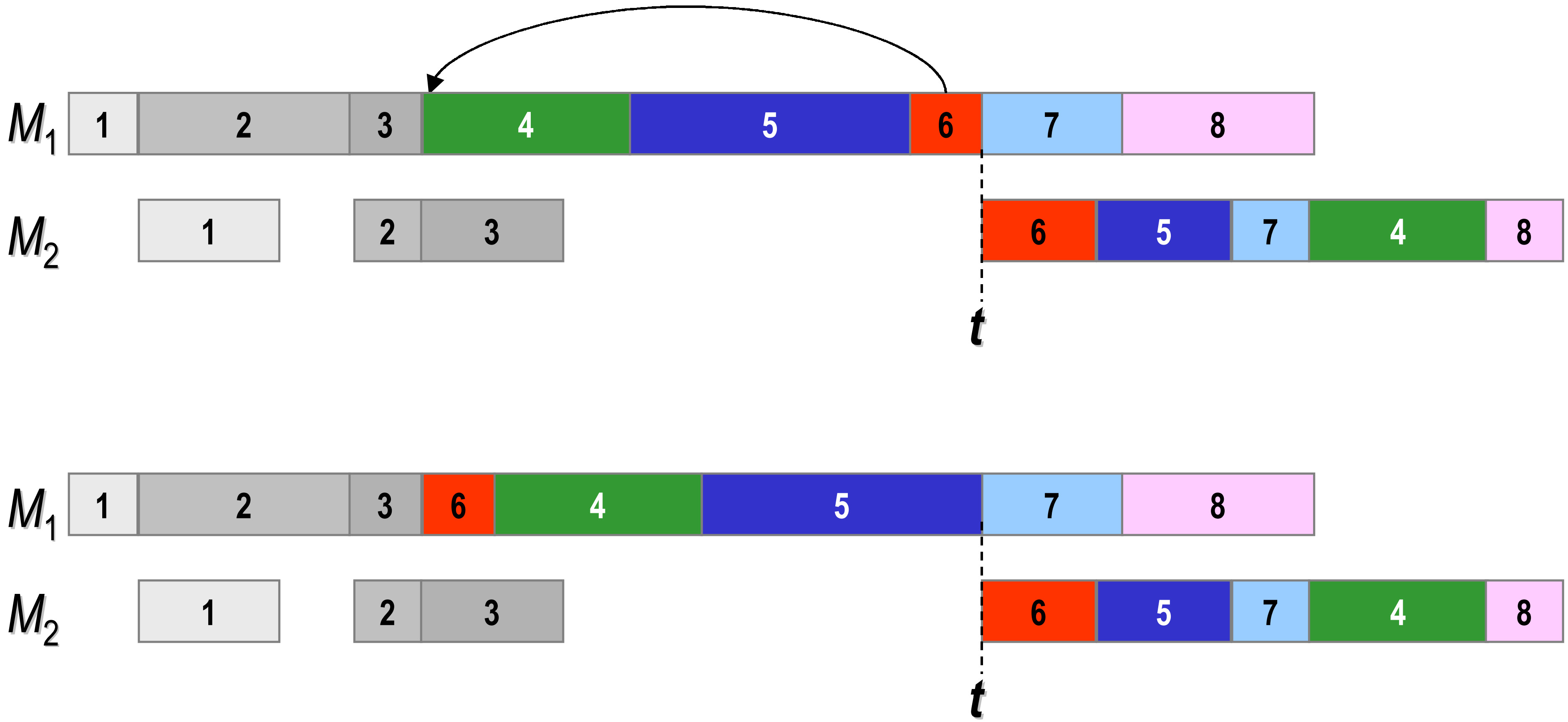
1. Single machine problems
    - Exact algorithms (SPT, SRPT, EDD)
    - Approximation algorithms
    - Branch and Bound
2. Parallel machine problems
    - Exact algorithms
    - Approximation algorithms
    - Branch and Bound
3. **Shop problems** $F \,||\, C_{max}$
    - Exact algorithms
    - Approximation algorithms
    - Branch and Bound

For problem $F2 || C_{max}$ there exists an optimal schedule with the same job sequence on the both machines.



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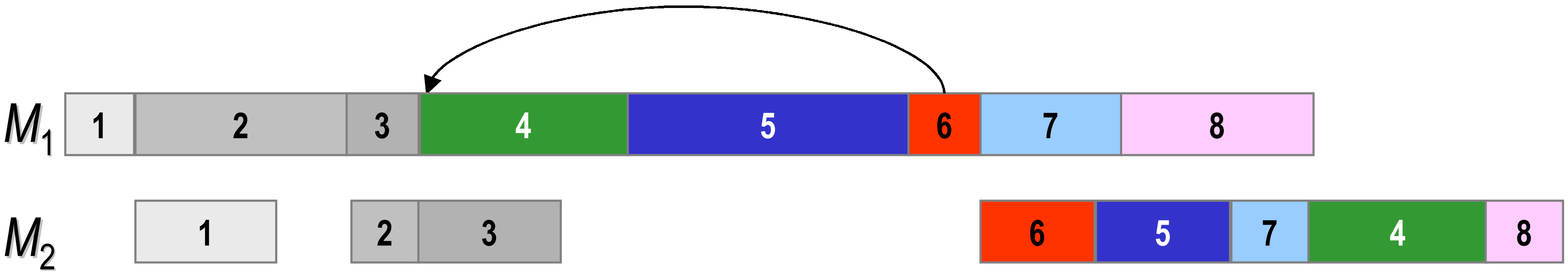

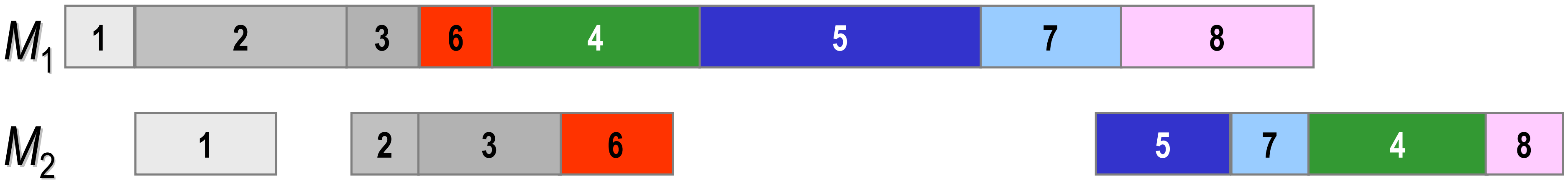

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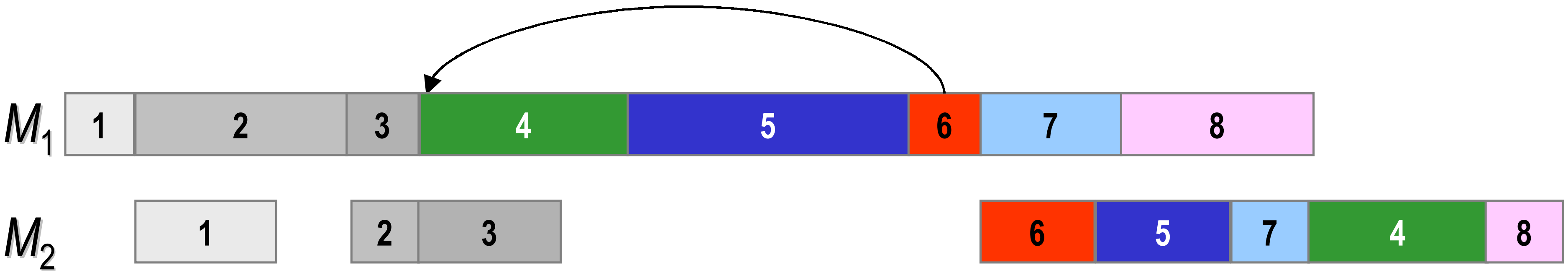

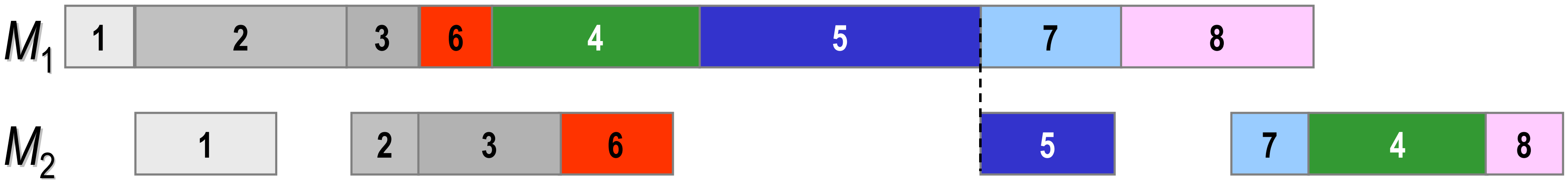

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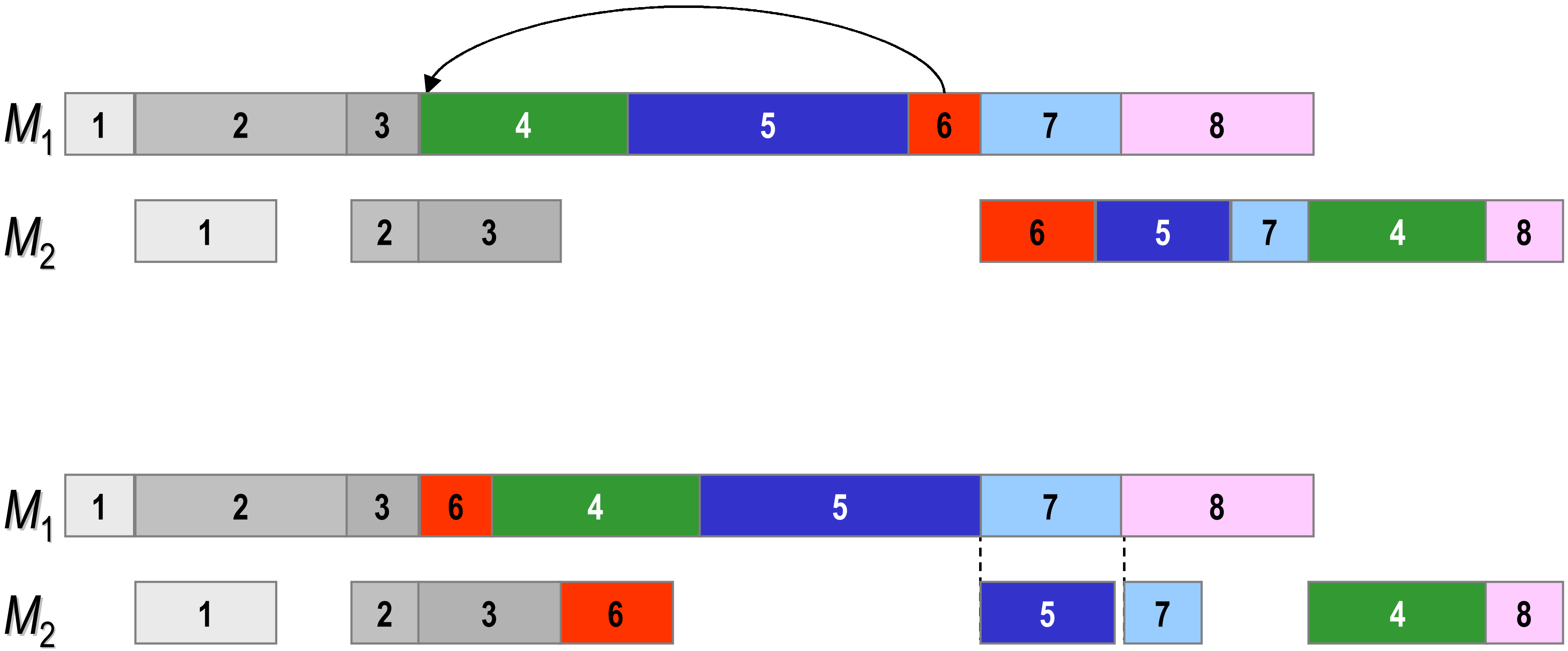

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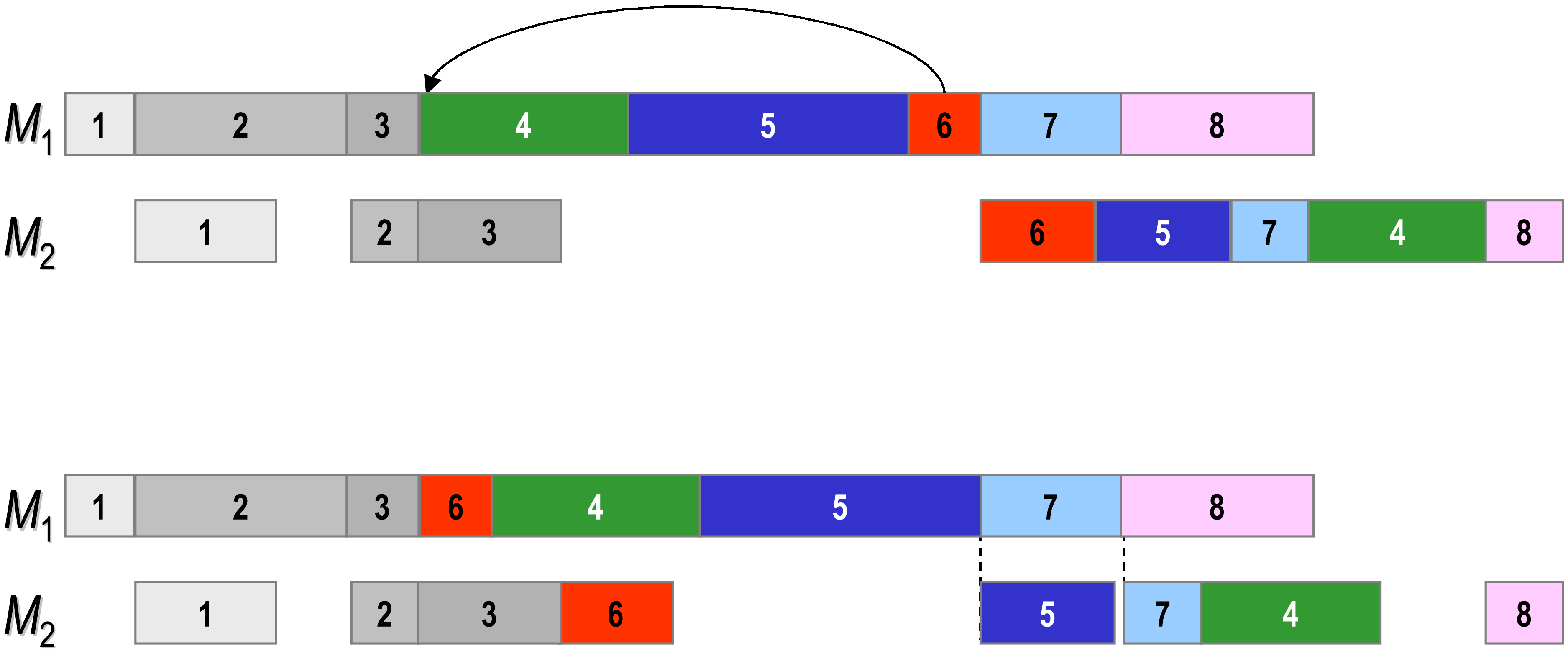

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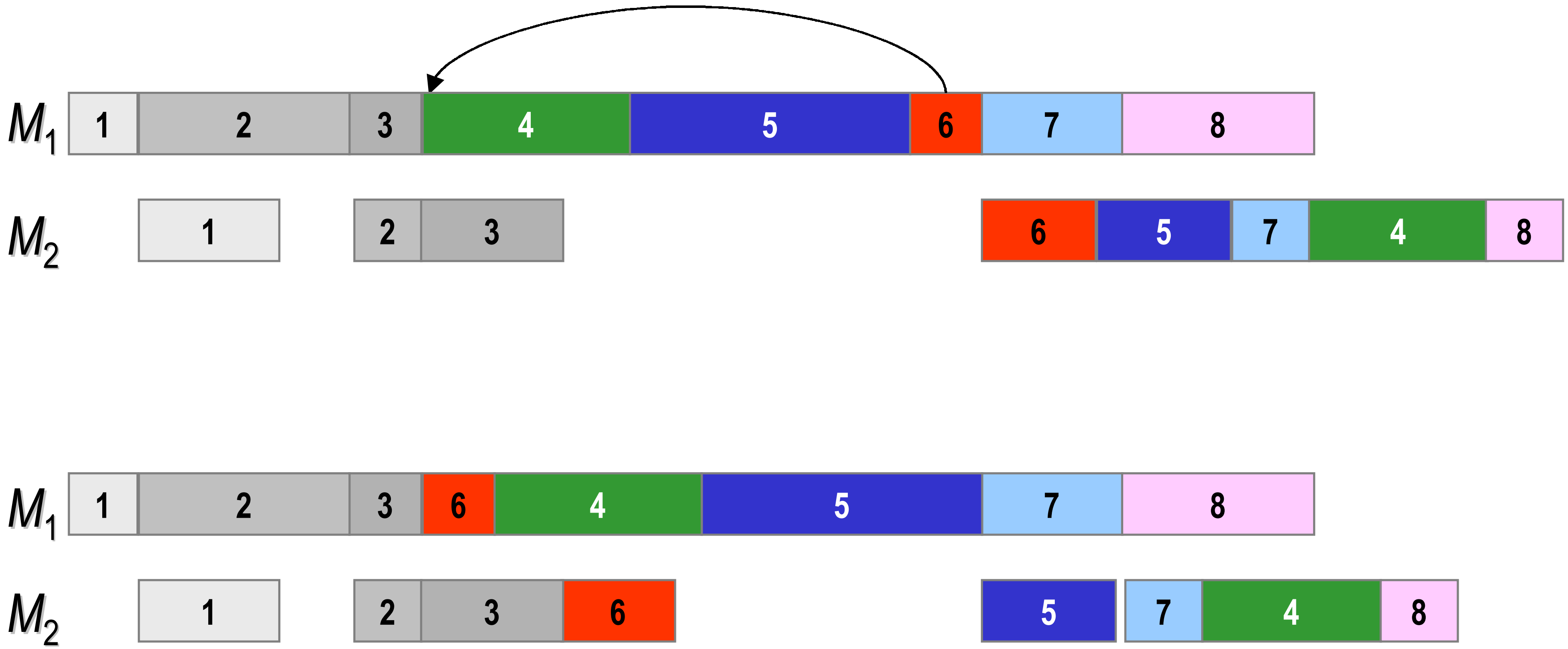

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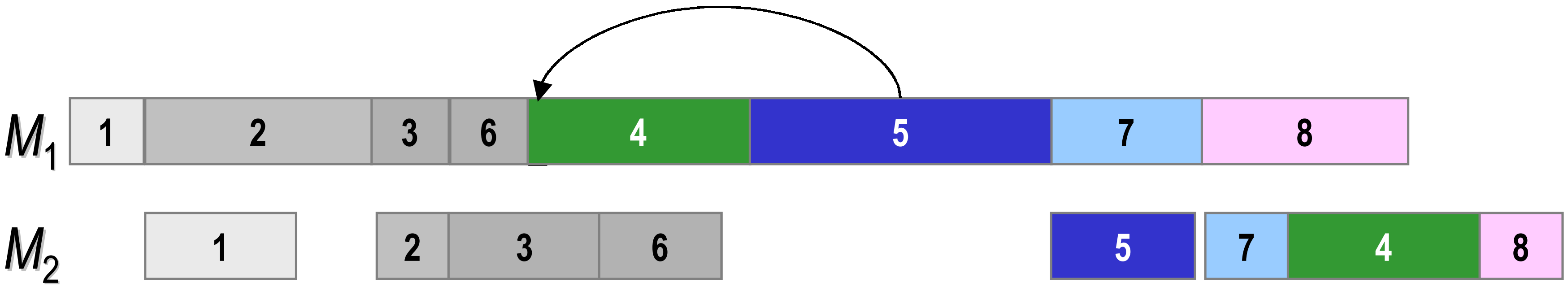

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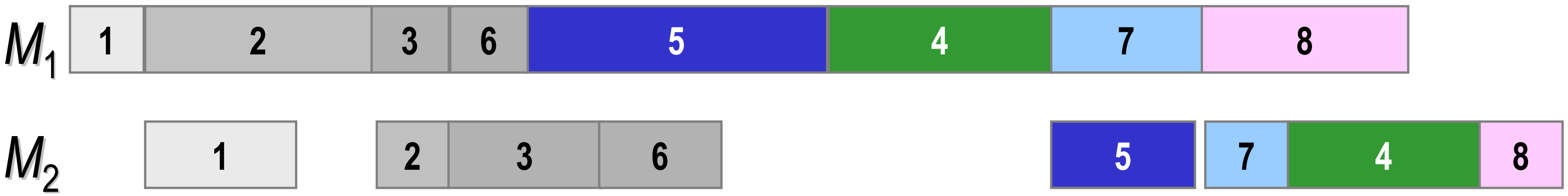

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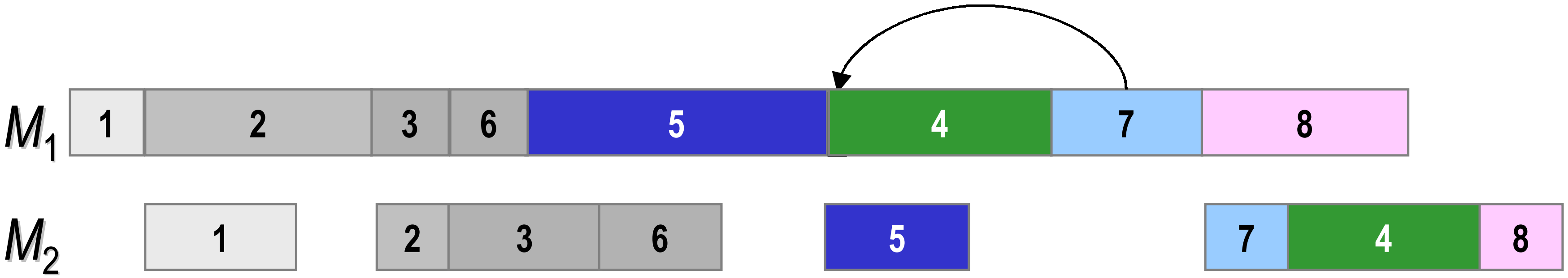

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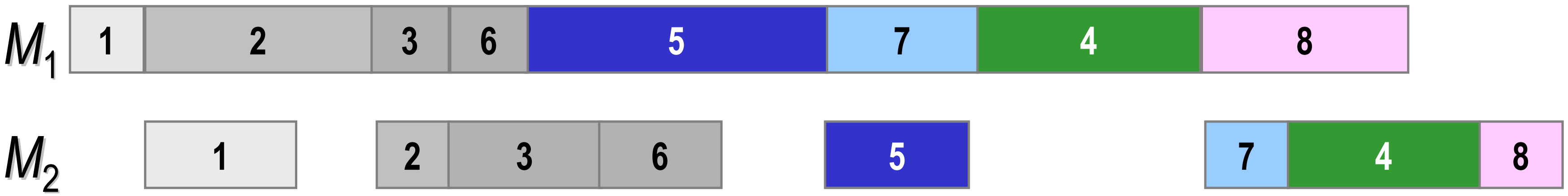

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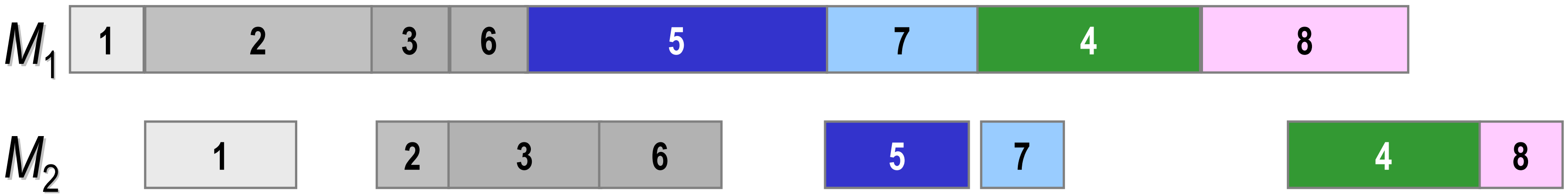

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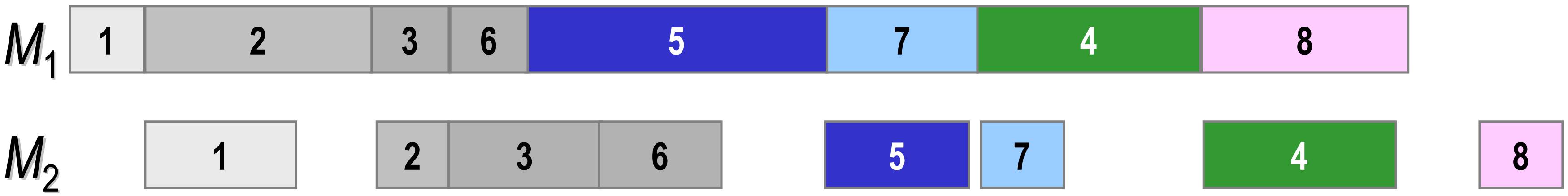

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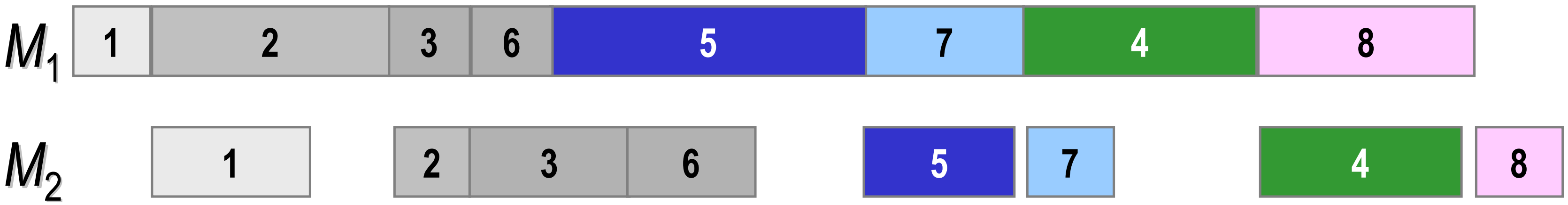
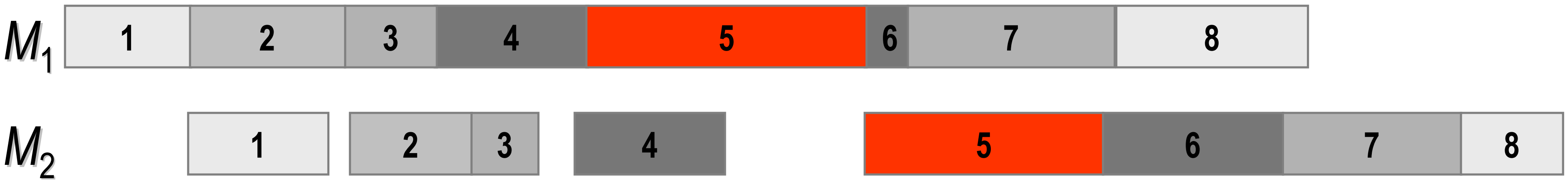
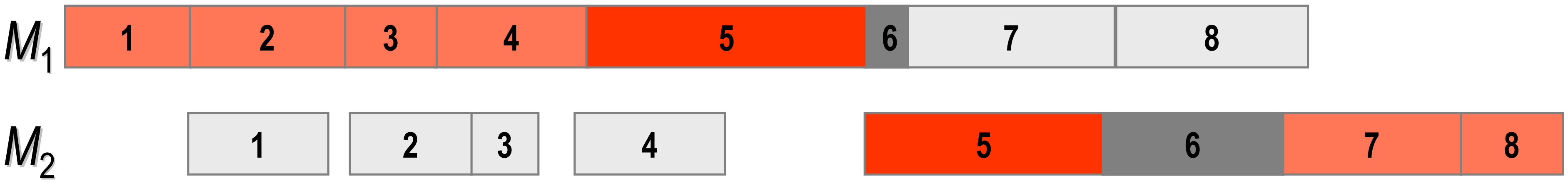


Johnson's algorithm



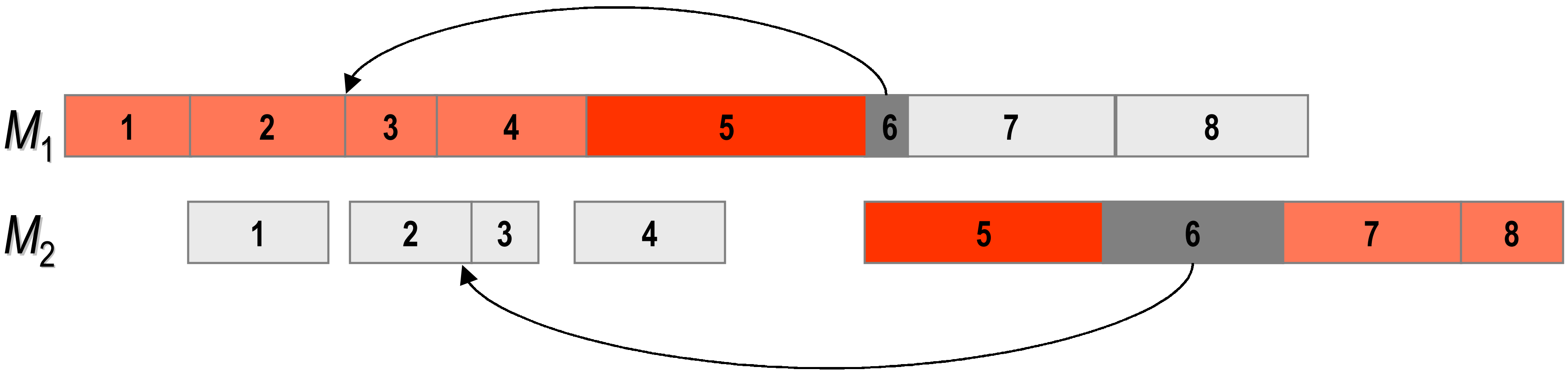
Johnson's algorithm

$$C_{max} = (p_{11} + p_{12} + p_{13} + p_{14} + p_{15}) + (p_{25} + p_{26} + p_{27} + p_{28})$$



Johnson's algorithm

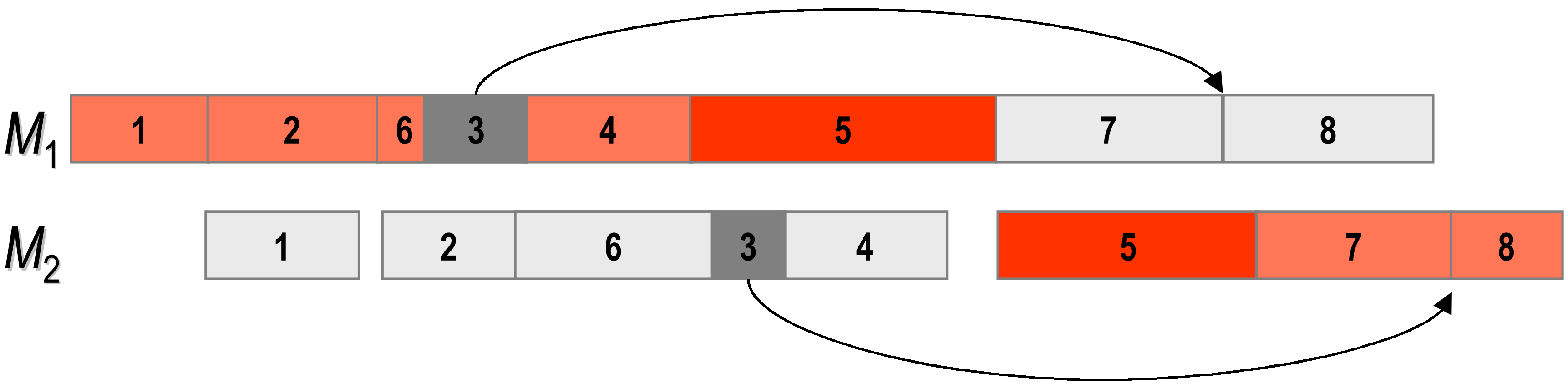
$$C_{max} = (p_{11} + p_{12} + p_{13} + p_{14} + p_{15}) + (p_{25} + p_{26} + p_{27} + p_{28})$$



Johnson’s algorithm

$$C_{max} = (p_{11} + p_{12} + p_{13} + p_{14} + p_{15}) + (p_{25} + p_{26} + p_{27} + p_{28})$$

$$C'_{max} = (p_{11} + p_{12} + p_{13} + p_{14} + p_{15} + p_{16}) + (p_{25} \qquad + p_{27} + p_{28})$$

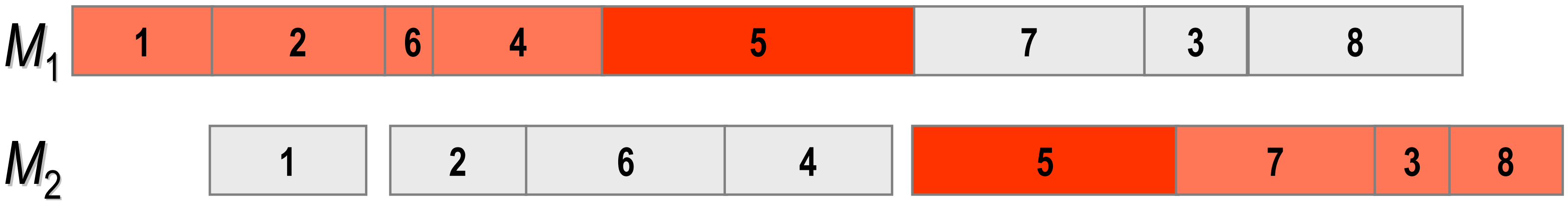


Johnson's algorithm

$$C_{max} = (p_{11} + p_{12} + p_{13} + p_{14} + p_{15}) + (p_{25} + p_{26} + p_{27} + p_{28})$$

$$C'_{max} = (p_{11} + p_{12} + p_{13} + p_{14} + p_{15} + p_{16}) + (p_{25} \quad + p_{27} + p_{28})$$

$$C''_{max} = (p_{11} + p_{12} \quad + p_{14} + p_{15} + p_{16}) + (p_{25} \quad + p_{27} + p_{23} + p_{28})$$



Johnson's algorithm

Johnson's algorithm constructs an optimal permutation schedule:

- Partition the jobs into two sets with set N_1 containing all the jobs with $p_{1j} < p_{2j}$ and set N_2 all the jobs with $p_{1j} \geq p_{2j}$.
- The jobs from set N_1 go first, and they go in increasing order of p_{1j} (SPT).
- The jobs from set N_2 follow in decreasing order of p_{2j} (LPT).

Example:

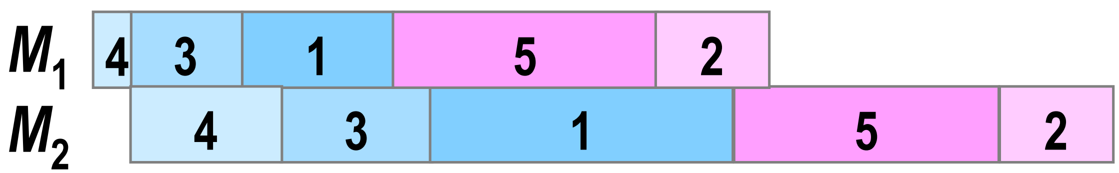
	M_1	M_2
J_1	4	8
J_2	3	3
J_3	3	4
J_4	1	4
J_5	8	7

SPT(M_1): J_4, J_3, J_1, LPT(M_2): J_5, J_2

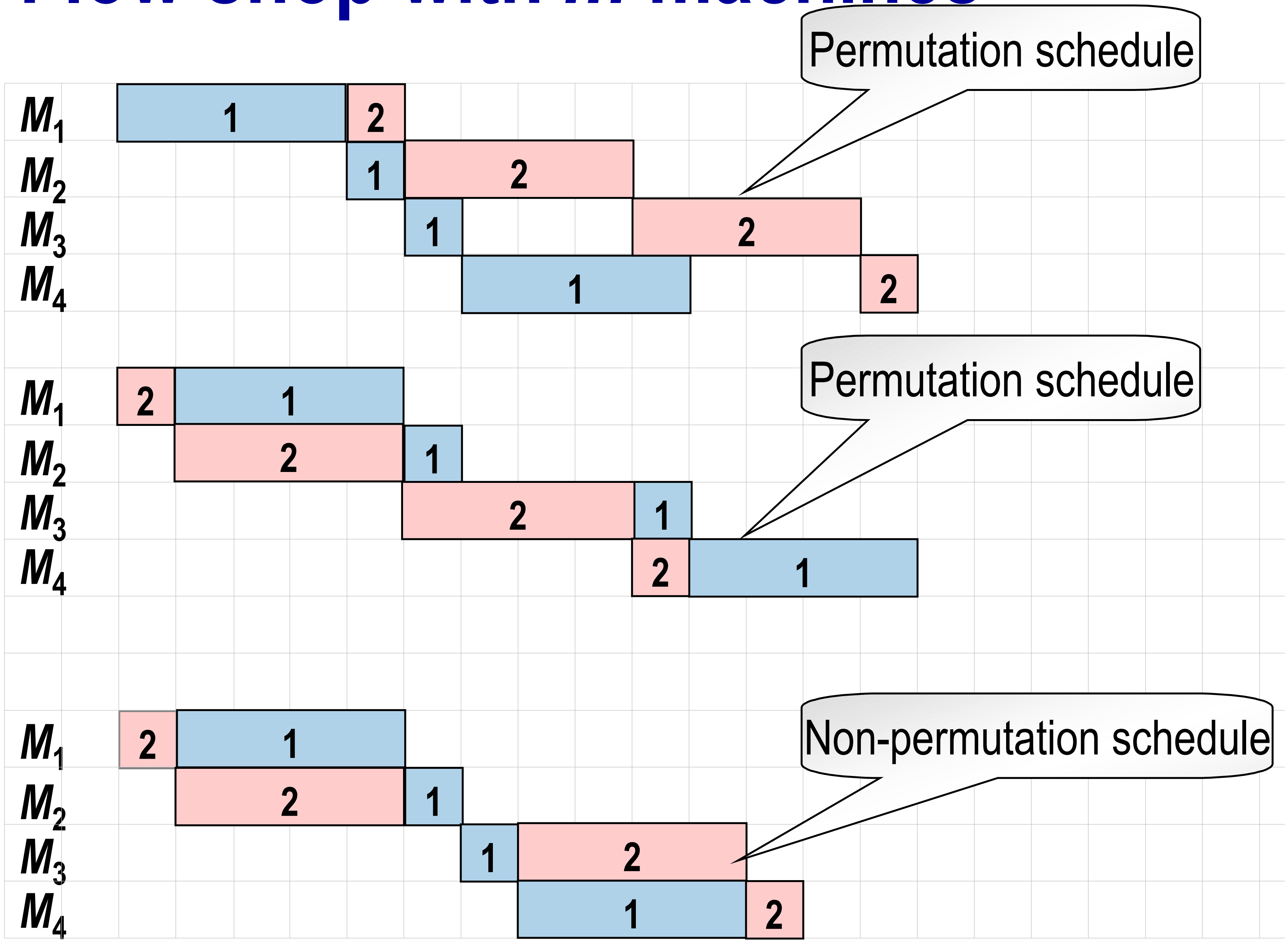
Johnson's algorithm

	M_1	M_2
J_1	4	8
J_2	3	3
J_3	3	4
J_4	1	4
J_5	8	7

SPT(M_1): J_4, J_3, J_1, LPT(M_2): J_5, J_2



Flow shop with *m* machines



Approximation algorithms for $F||C_{max}$

Decomposition Algorithm

	M_1	M_2	M_3	M_4
J_1	5	4	4	3
J_2	5	4	4	6
J_3	3	2	3	3
J_4	6	4	4	2
J_5	3	4	1	5

Approximation algorithms for $F||C_{max}$

Decomposition Algorithm *FSDecomp*

	M_1	M_2	M_3	M_4
J_1	5	4	4	3
J_2	5	4	4	6
J_3	3	2	3	3
J_4	6	4	4	2
J_5	3	4	1	5

SPT(M_1): LPT(M_2)

Approximation algorithms for $F||C_{max}$

Decomposition Algorithm *FSDecomp*

	M_1	M_2	M_3	M_4
J_1	5	4	4	3
J_2	5	4	4	6
J_3	3	2	3	3
J_4	6	4	4	2
J_5	3	4	1	5

SPT($\boldsymbol{M}_1$): J_5, LPT($\boldsymbol{M}_2$) J_1, J_2, J_4, J_3.

SPT($\boldsymbol{M}_3$): J_5, LPT($\boldsymbol{M}_4$) J_2, J_1, J_3, J_4.

Job sequence on M_1, M_2: J_5, J_1, J_2, J_4, J_3.

Job sequence on M_3, M_4: J_5, J_2, J_1, J_3, J_4.

Approximation algorithms for $F||C_{max}$

Aggregation Algorithm *FSAggr*

	M_1	M_2	M_3	M_4
J_1	5	4	4	3
J_2	5	4	4	6
J_3	3	2	3	3
J_4	6	4	4	2
J_5	3	4	1	5

	A	B
J_1	9	7
J_2	9	10
J_3	5	6
J_4	10	6
J_5	7	6

SPT(***A***): J_3, J_2, LPT(***B***) J_1, J_4, J_5

Job sequence on M_1, M_2, M_3, M_4: J_3, J_2, J_1, J_4, J_5.

List of the results

Problem	Algorithm
$F2 \| \| C_{max}$	**Johnson's rule** $SPT(M_1) - LPT(M_2)$
$F \| \| C_{max}$	**Decomposition** Gonzalez & Sahni (1978) $\lceil m/2 \rceil$ - approximation
$F \| \| C_{max}$	**Aggregation** Röck & Schmidt (1982) $\lceil m/2 \rceil$ - approximation
$F \| \| \Sigma C_j$	**SPT** $\lceil m \rceil$ - approximation