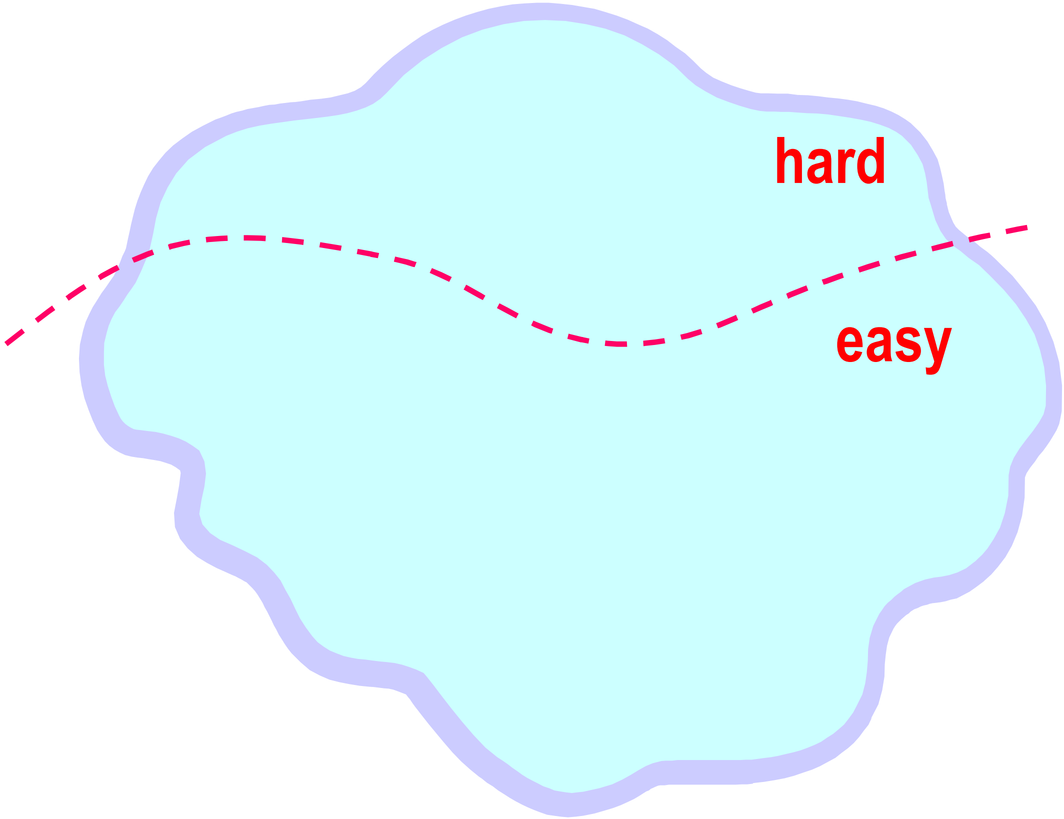
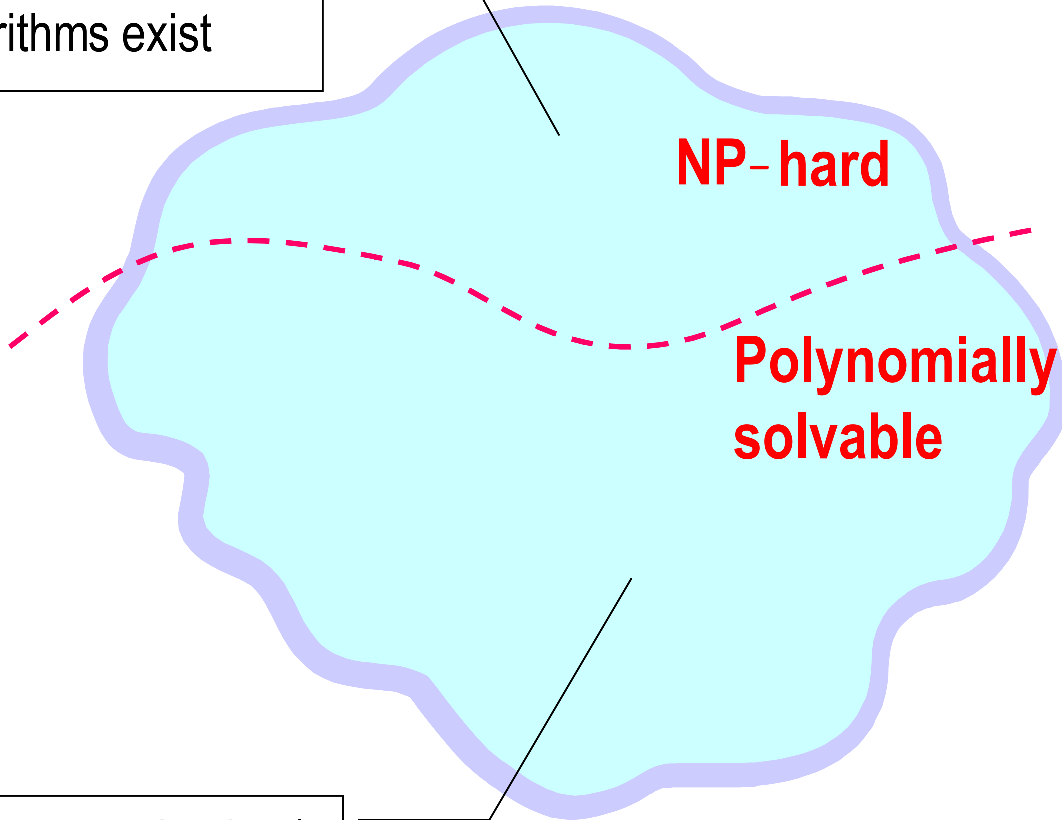
Computational Complexity



Computational Complexity

- “intractable”,
- unlikely that polynomial-time algorithms exist



- efficient (polynomial time) algorithms exist

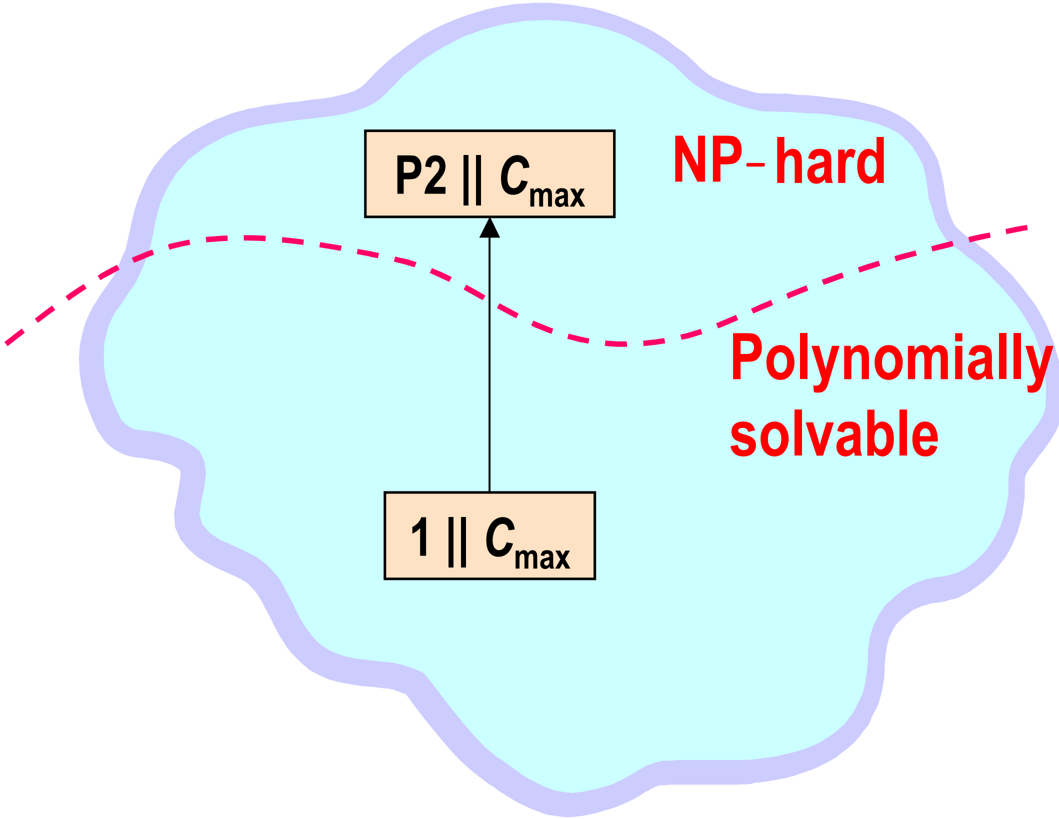
Running time is bounded by a polynomial in input size:

$O(n^2)$ - the number of steps grows as Cn^2

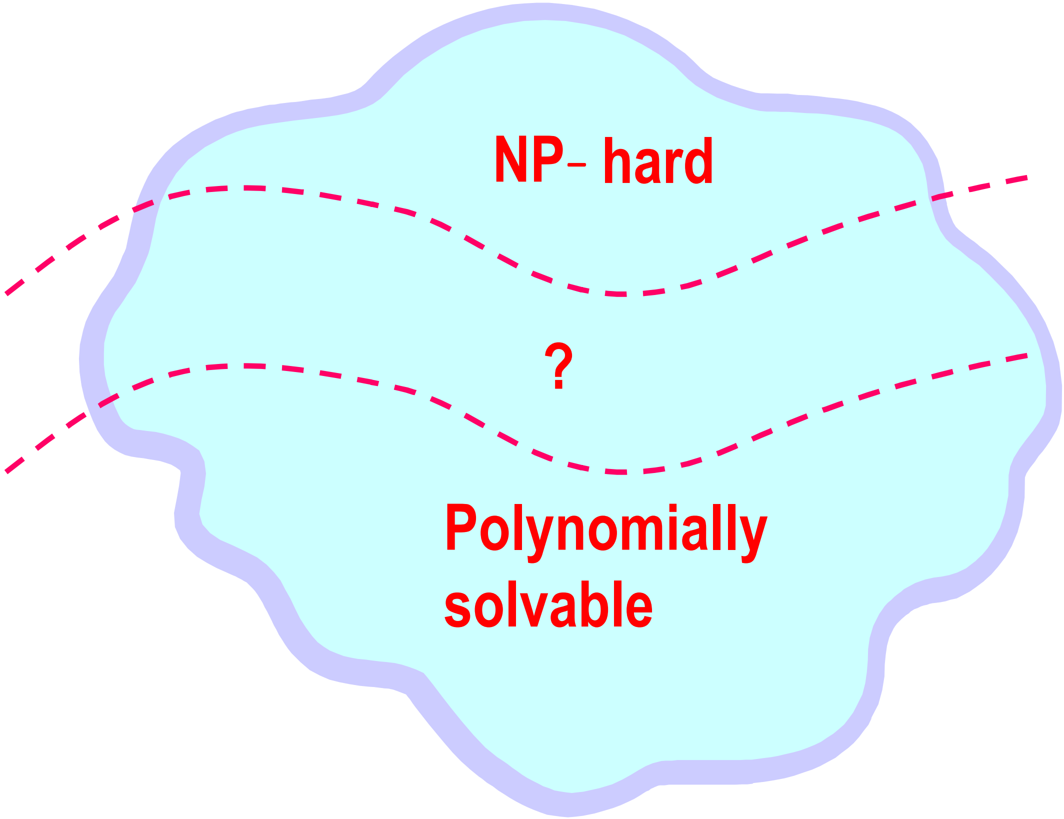
$O(nm)$ - the number of steps grows as Cnm

$O(n \log n)$ - the number of steps grows as $Cn\log n$

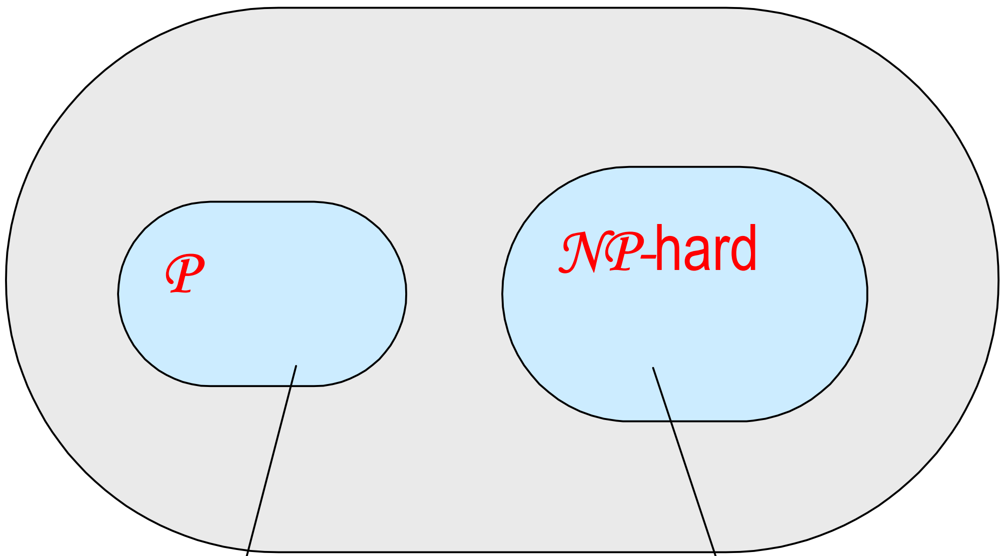
Computational Complexity



Computational Complexity



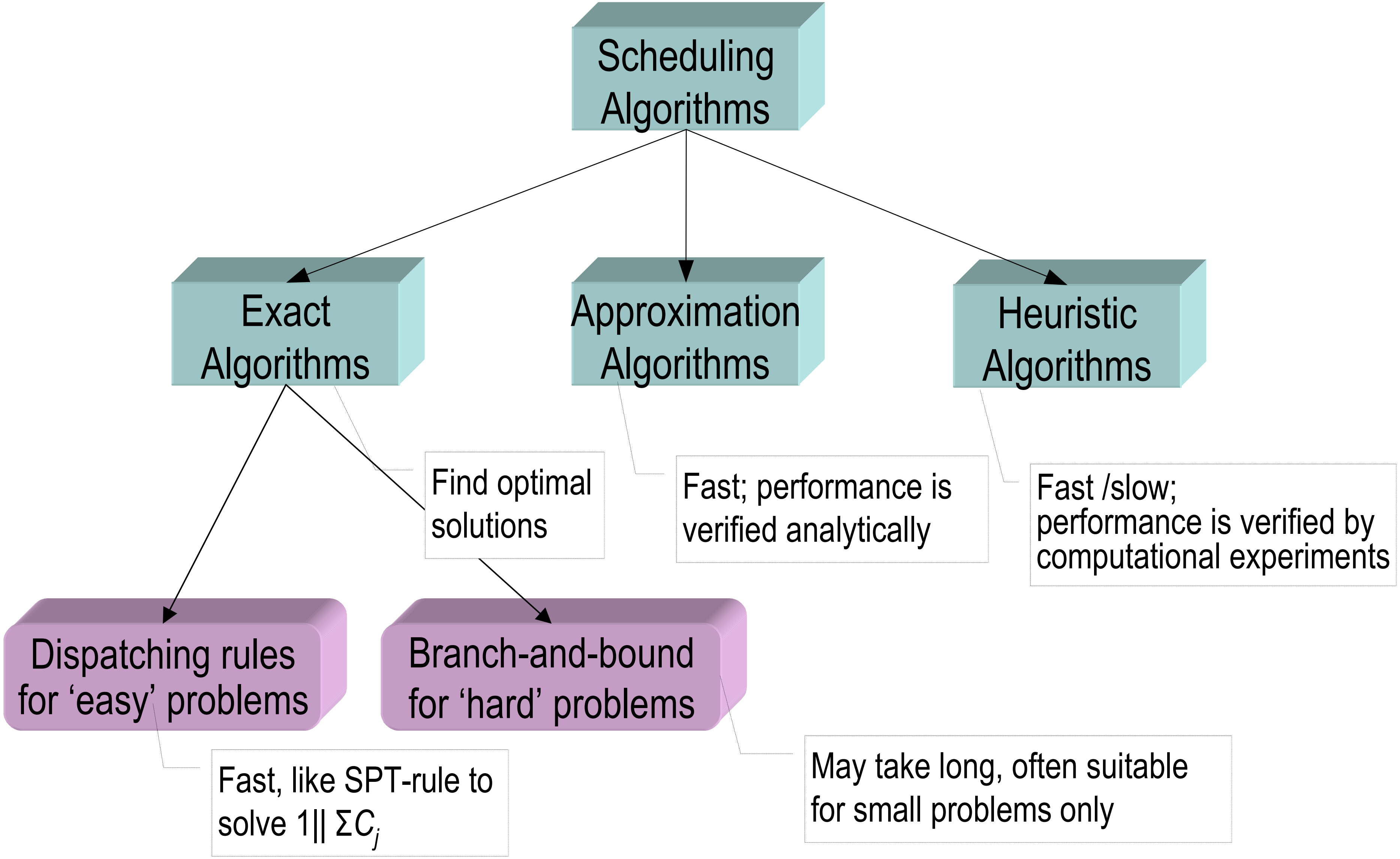
Computational Complexity



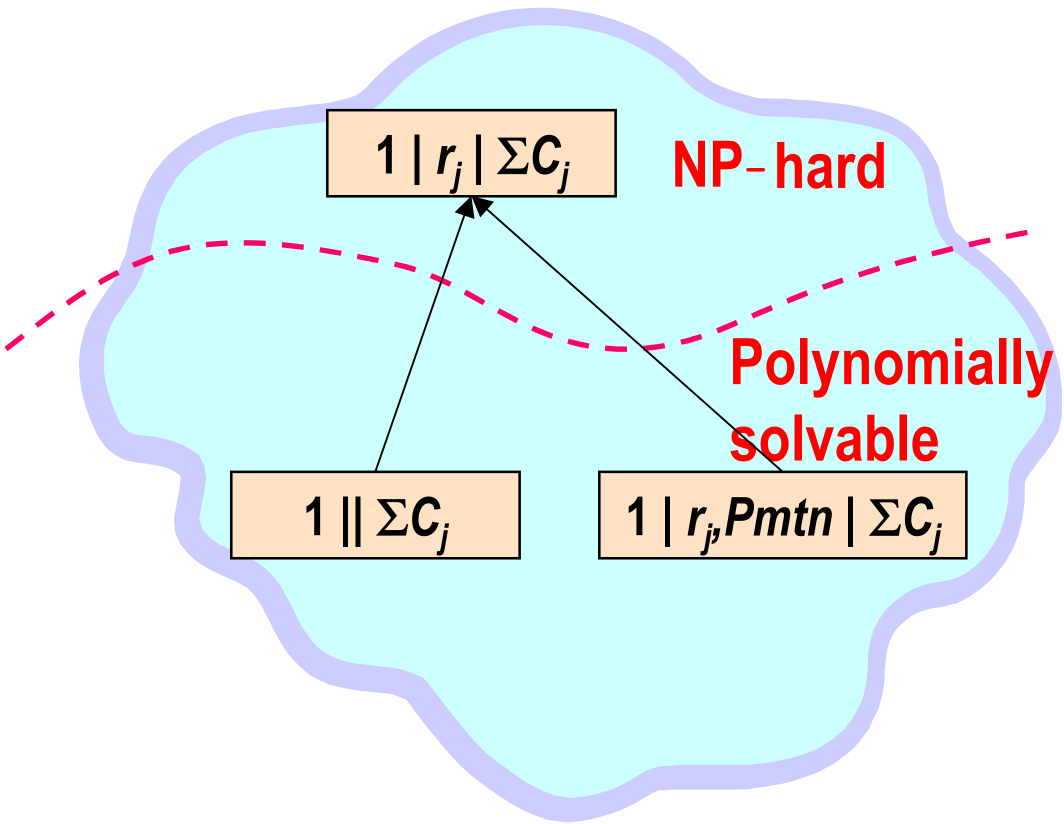
- efficient (polynomial time) algorithms exist

- “intractable”,
- unlikely that a polynomial-time algorithm exists

Living with NP-hard Problems



Problem $1 | r_j | \Sigma C_j$



$1|\, r_j\,,\, \mathrm{Pmtn}|\, \Sigma C_j$ and $1|\, r_j \,|\, \Sigma C_j$

SRPT
Schrage (1965)

2-approximation
Philips, Stein & Wein (1995)

Algorithm A (*modification of the SRPT algorithm)*

1. Solve the preemptive version of the problem using SRPT-rule.
2. Sequence the jobs non-preemptively in the order that they complete in the solution of the preemptive problem.

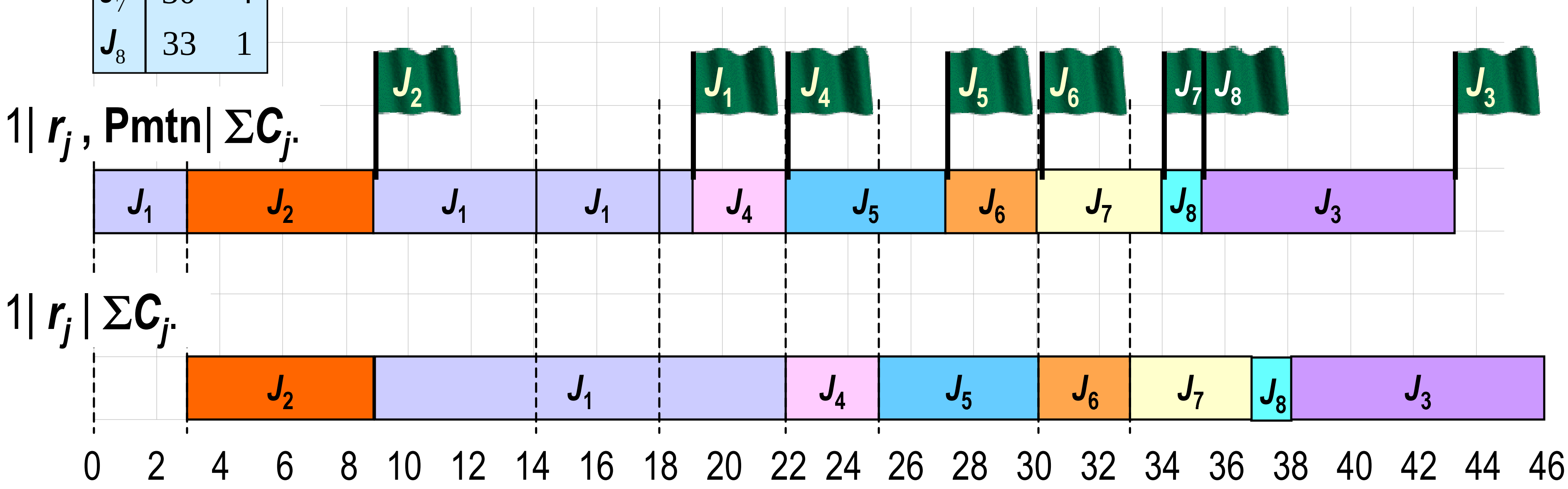
$1|\ r_j\ ,\ \mathrm{Pmtn}|\ \Sigma C_j$ and $1|\ r_j\ |\ \Sigma C_j$

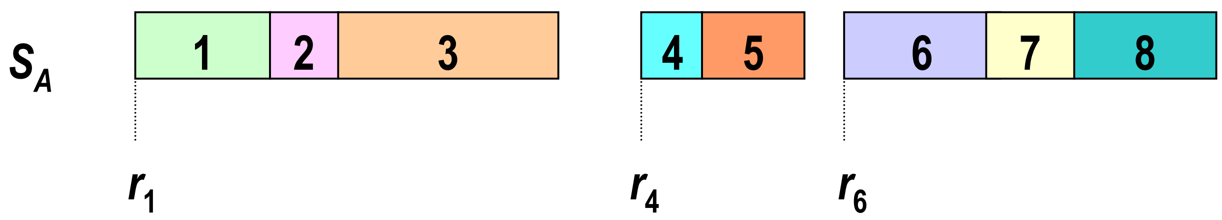
	r_j	p_j
J_1	0	13
J_2	3	6
J_3	14	8
J_4	18	3
J_5	22	5
J_6	25	3
J_7	30	4
J_8	33	1

$F(S_A) = 240$

$F(S_{OPT}) = 223$

Actual accuracy = 240/223 = 1.08 (or 8 %)





Theorem 1. For $1|r_j|\sum C_j$, $\dfrac{F(S_A)}{F(S_{OPT})} \le 2$

Proof. Let us renumber the jobs in the order they are completed in $\boldsymbol{S_A}$. Then the completion time of each job $\boldsymbol{j}$ in schedule $\boldsymbol{S_A}$ is

$$C_j(S_A) = r_u + \sum_{k=u}^{j} p_k \qquad \textbf{(1)}$$

where $\boldsymbol{u}$ is the nearest job that precedes job $\boldsymbol{j}$ and starts at $\boldsymbol{r_u}$.

Denote the completion time of job $\boldsymbol{j}$ in the preemptive schedule by $\boldsymbol{C_j}(S_{Pmtn})$.

$$\begin{cases} r_u < C_u(S_{Pmtn}) < C_j(S_{Pmtn}) \\ \sum_{k=u}^{j} p_k \le C_j(S_{Pmtn}) \end{cases} \Rightarrow \sum_{j=1}^{n} C_j(S_A) \le 2\sum_{j=1}^{n} C_j(S_{Pmtn}).$$

The observation that $\Sigma \boldsymbol{C_j}(S_{Pmtn})$ is a lower bound on the optimal value of the total completion time for the nonpreemptive schedule $\Sigma \boldsymbol{C_j}(S_{OPT})$ provides the ratio guarantee of 2.

■