Single Machine Problems: Due Date Scheduling

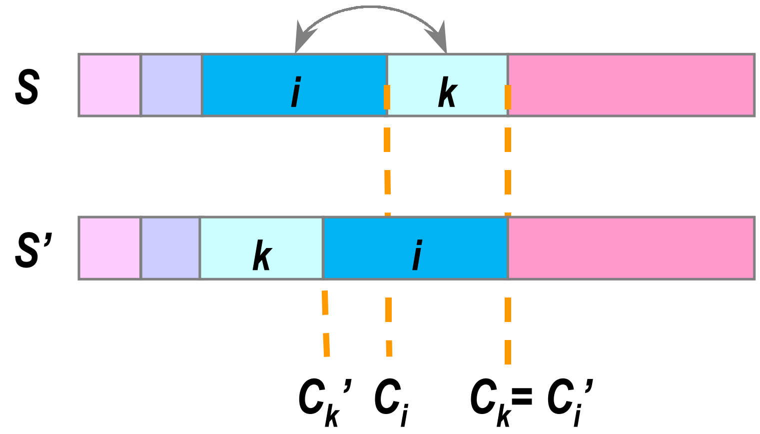
Previous lecture:

- $1 \mid\mid \Sigma w_j C_j$
- $1 \mid Pmtn \mid \Sigma C_j$
- $1 \mid r_j, Pmtn \mid \Sigma C_j$

This lectures:

- $1 \mid\mid L_{\max}$
- $1 \mid\mid \Sigma U_j$

1. Maximum Lateness



Consider **1| |** $\boldsymbol{L_{max}}$,

where $L_{max} = \max\{L_j = C_j - d_j \mid j=1,\ldots, n\}$.

Theorem 1. For 1|| $\boldsymbol{L_{max}}$ the EDD rule is optimal.

Proof. **Adjacent pairwise interchange.**

Suppose a schedule ***S***, which violates EDD, is optimal. In this schedule there must be at least two adjacent jobs ***i*** and ***k*** such that $\boldsymbol{d_i > d_k}$.

Swapping jobs ***j*** and ***k*** leads to a schedule ***S'*** such that

$$\max\{C_j' - d_j, C_k' - d_k\} =$$

$$\max\{C_k - d_j, C_k - p_j - d_k\} < \max\{C_k - d_k, C_k - p_j - d_k\} = C_k - d_k$$

$$= \max\{C_j - d_j, C_k - d_k\}.$$

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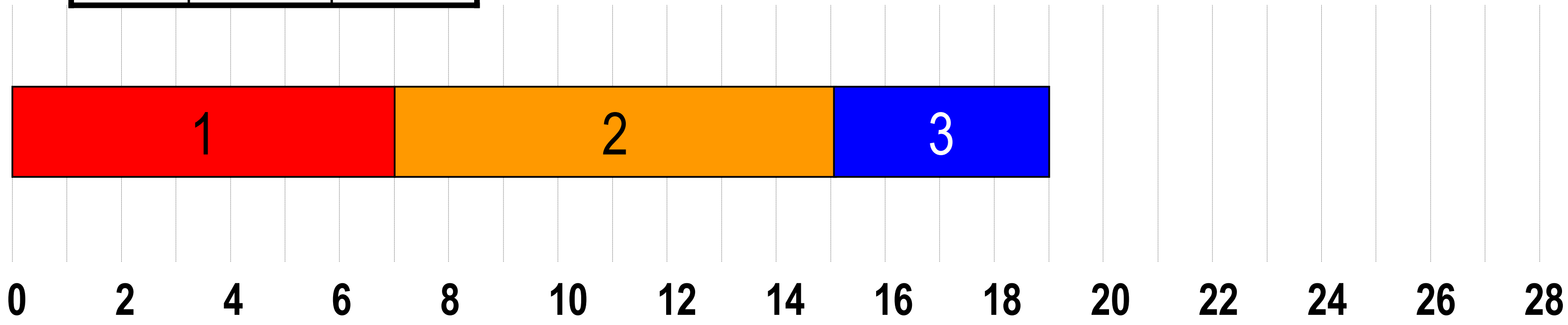
2. The number of tardy jobs: Moore's-rule

Consider $1||\sum U_j$, where $U_j = \begin{cases} 0, & \text{if } C_j \le d_j \\ 1, & \text{otherwise} \end{cases}$

Job	p_j	d_j
1	7	9
2	8	17
3	4	18
4	6	19
5	6	20



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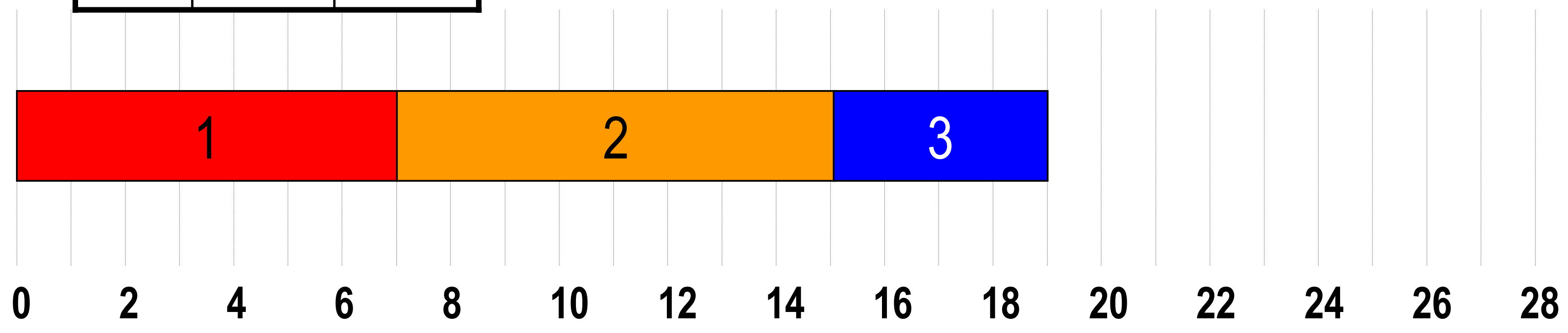
By removing the job with the largest processing time, we guarantee that the total processing time of on-time jobs is as small as possible !



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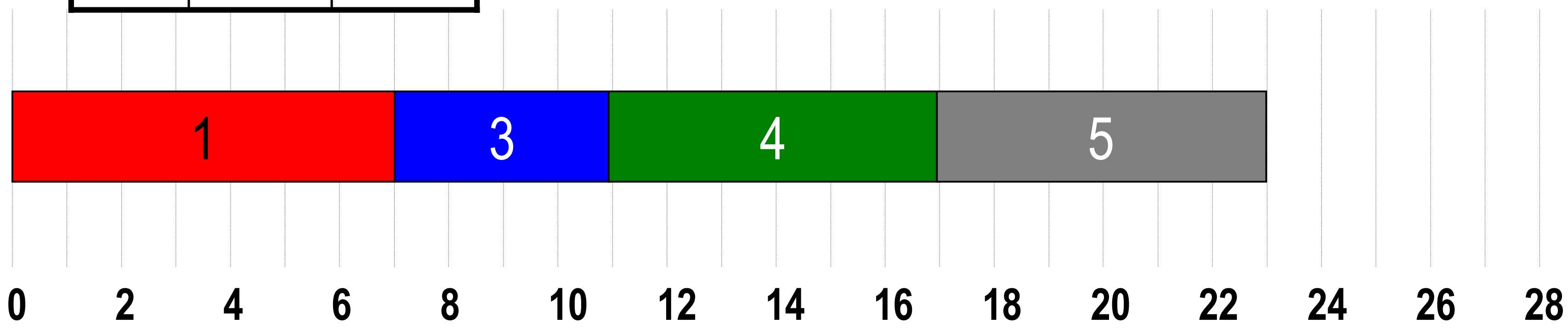

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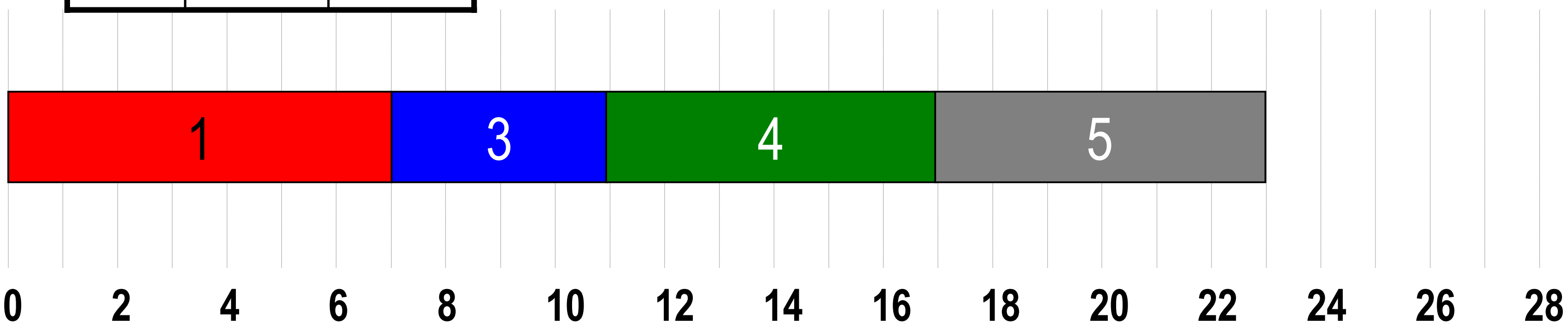

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Moore's algorithm:

- The algorithm repeatedly adds jobs in the EDD order to the end of a partial schedule of on-time jobs.
- If the addition of job j results in this job being completed after time $\boldsymbol{d_j}$, then a job in the partial schedule with the largest processing time is removed and declared late.
- All late jobs are scheduled in an arbitrary order after on-time jobs.