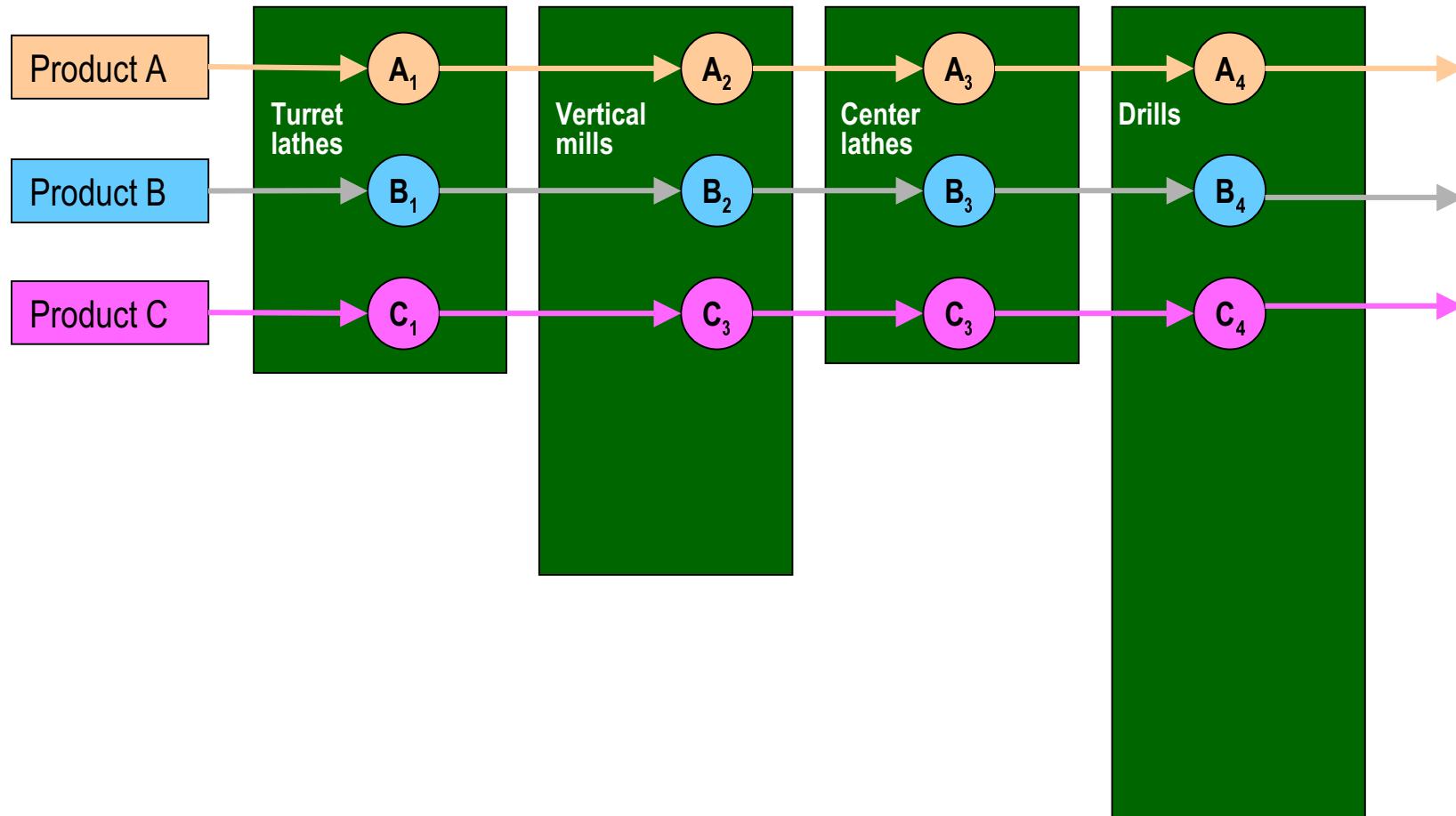
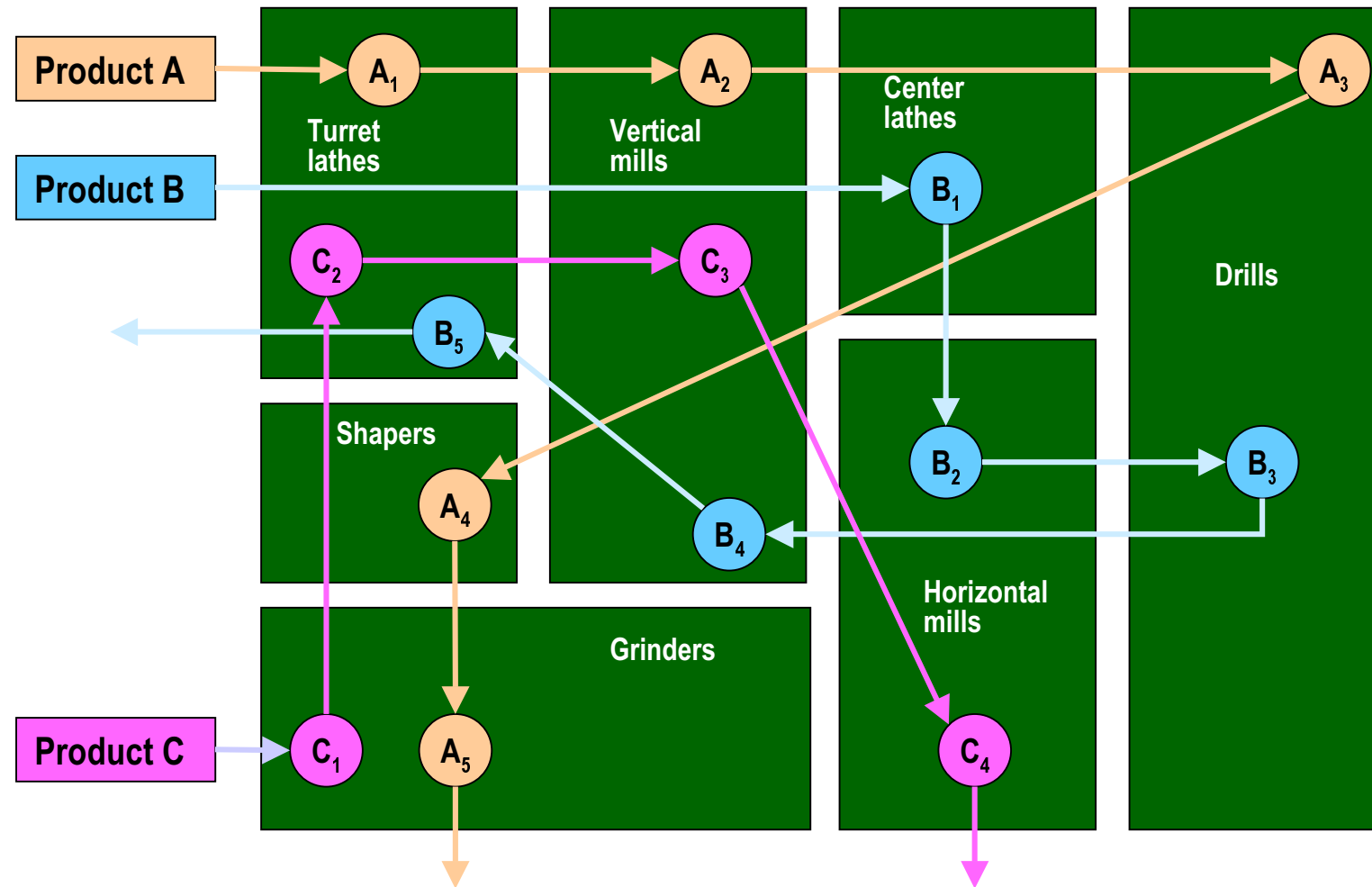


Flow Shop



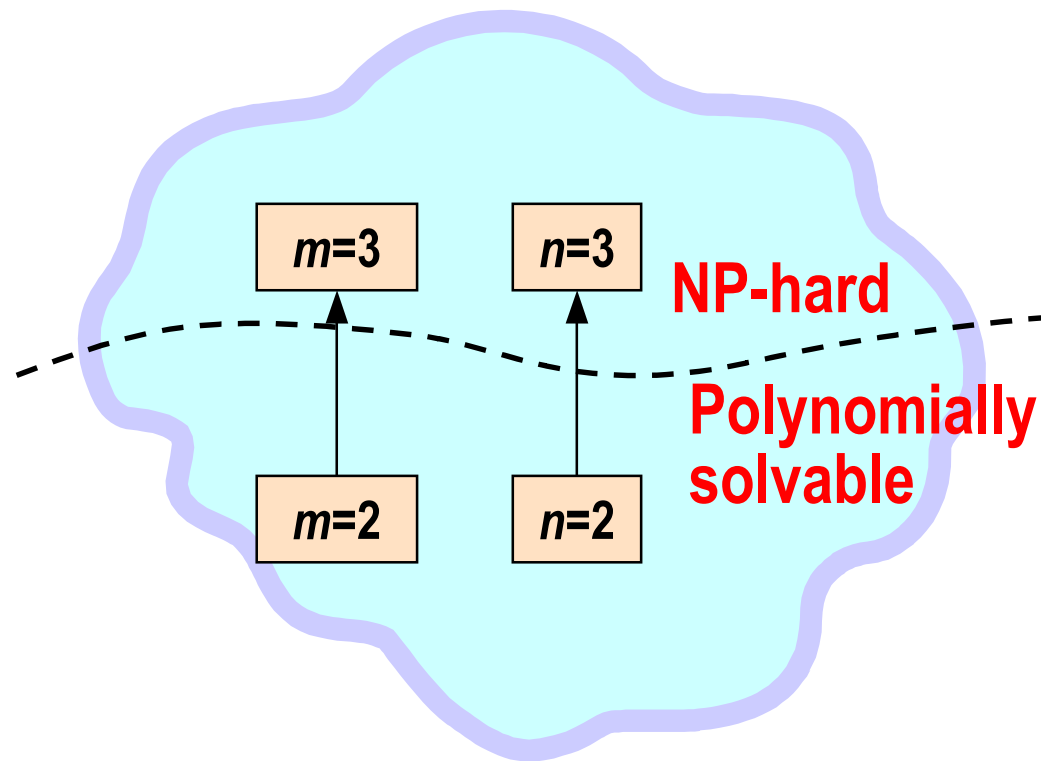
From "Fundamentals of Operations Management"
by Davis, Aquilano, Chase (1999)

Job Shop



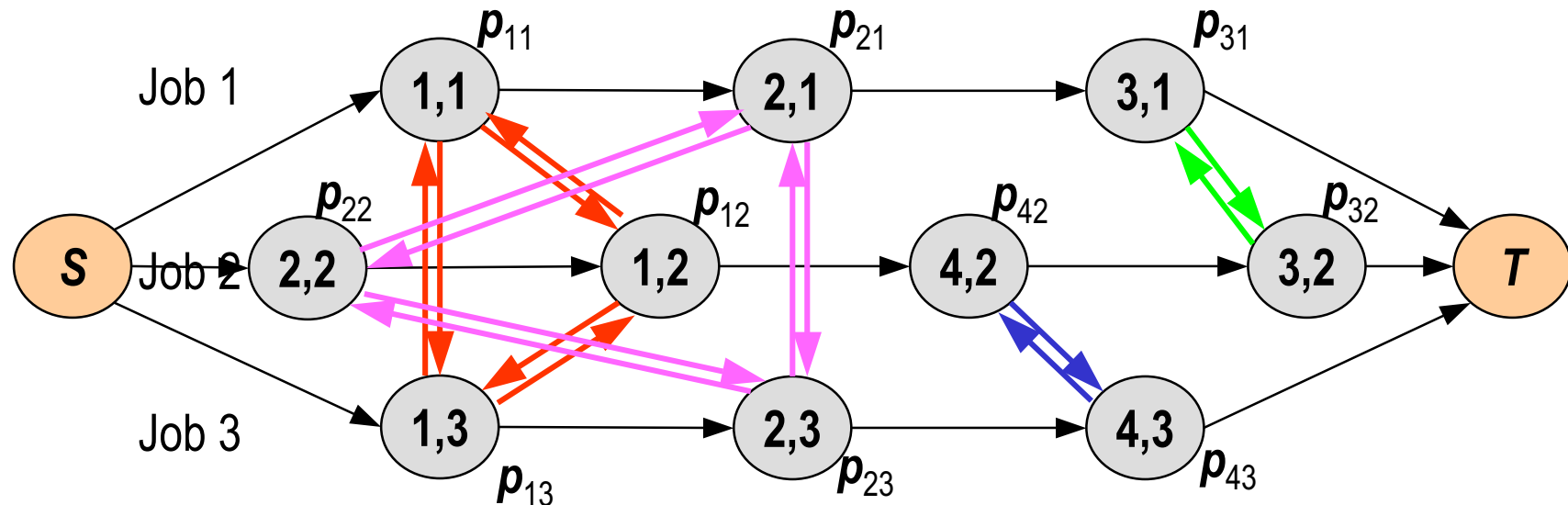
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Complexity



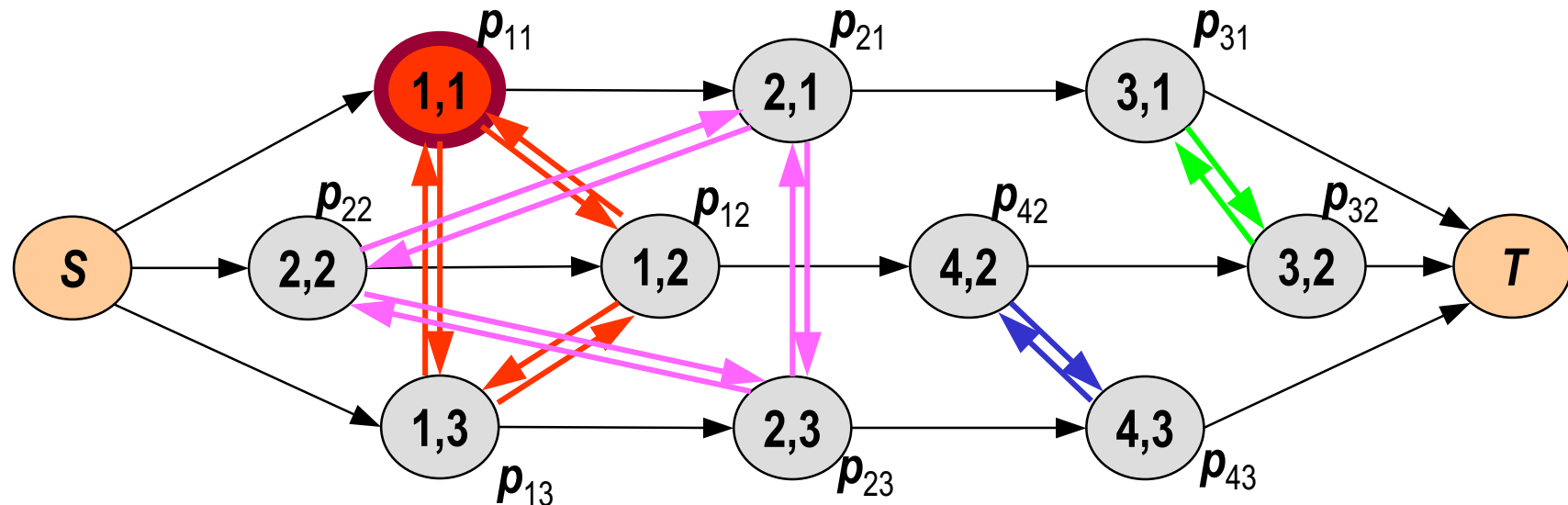
Job Shop: Disjunctive Graph Model

Jobs	Machine sequence	Processing times
1	1, 2, 3	$p_{11}=10, p_{21}=8, p_{31}=4$
2	2, 1, 4, 3	$p_{12}=3, p_{22}=8, p_{32}=6, p_{42}=5$
3	1, 2, 4	$p_{13}=4, p_{23}=7, p_{43}=3$



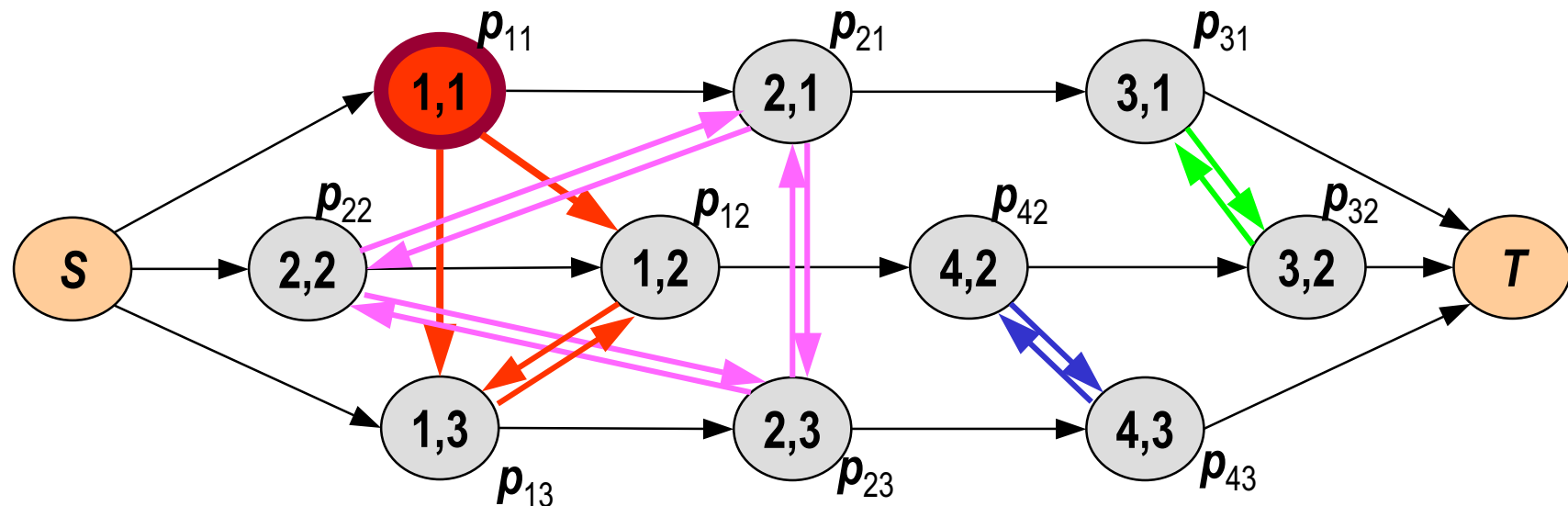
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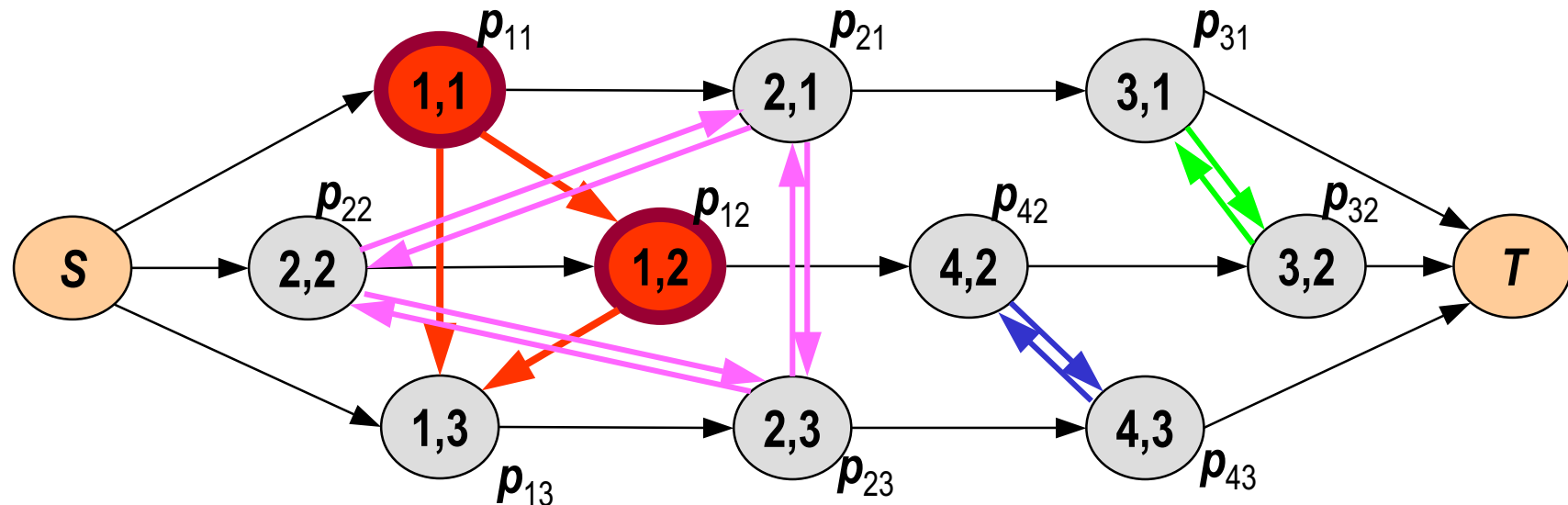
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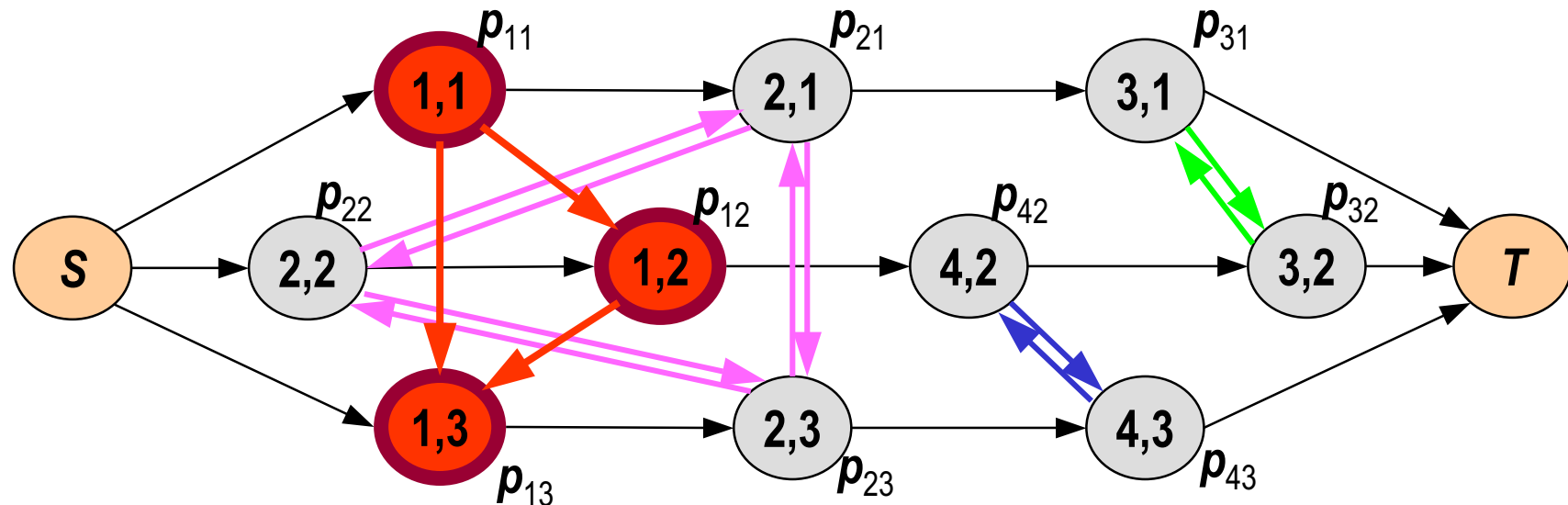
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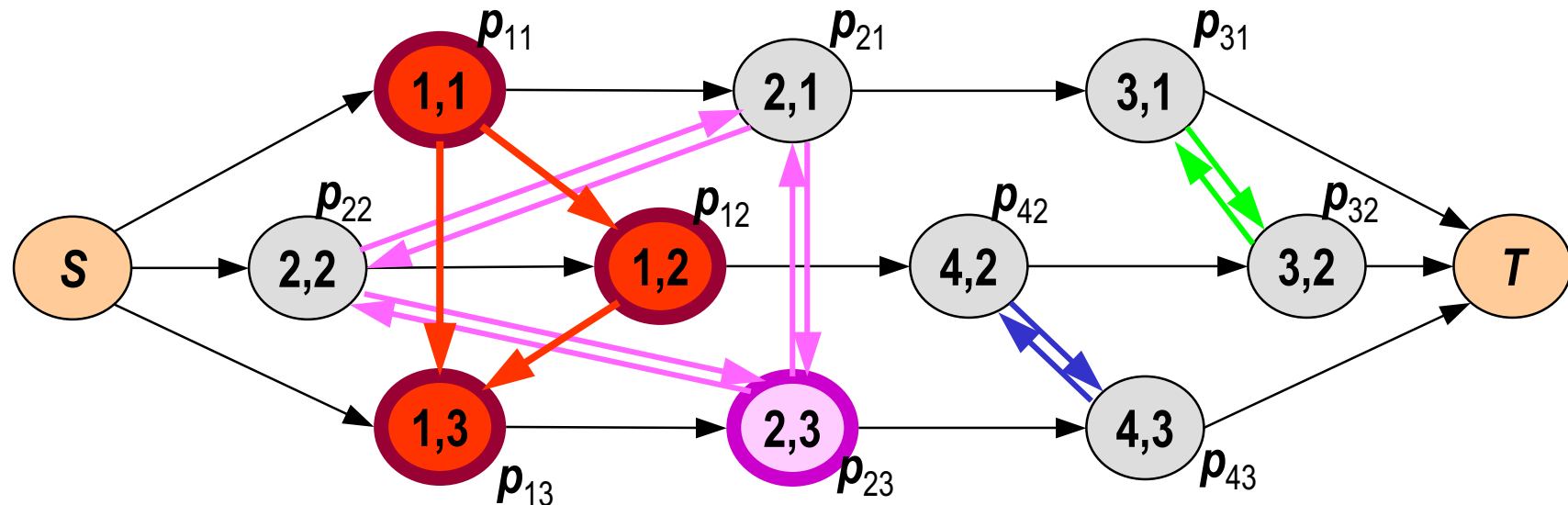
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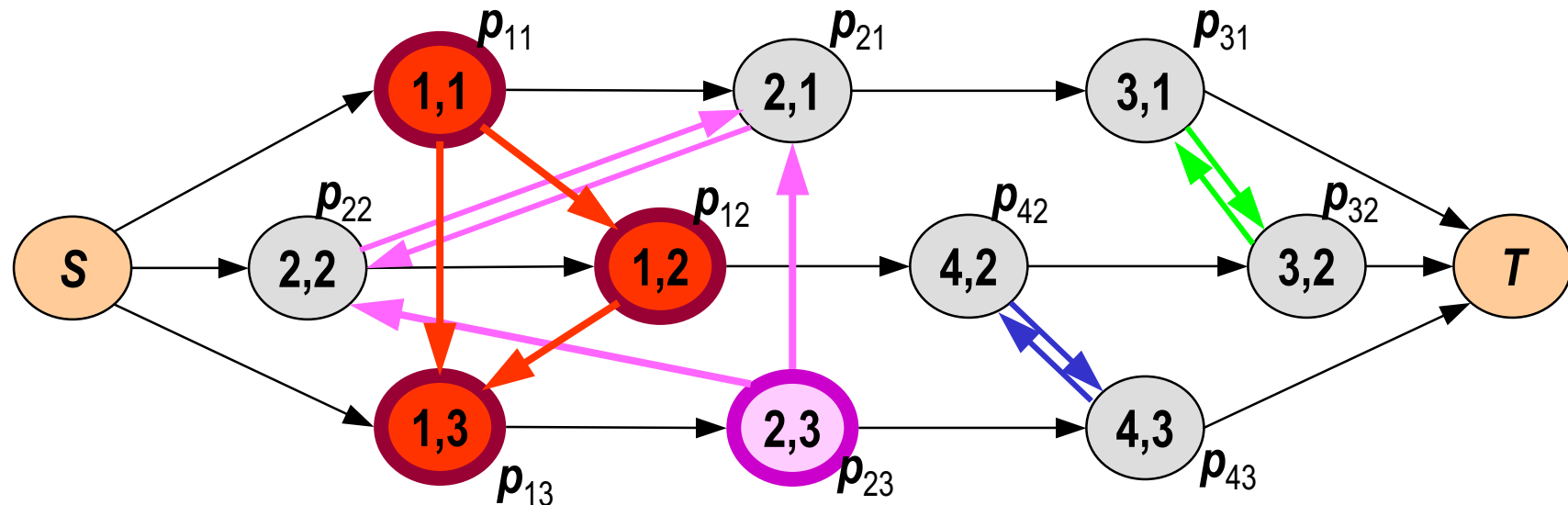
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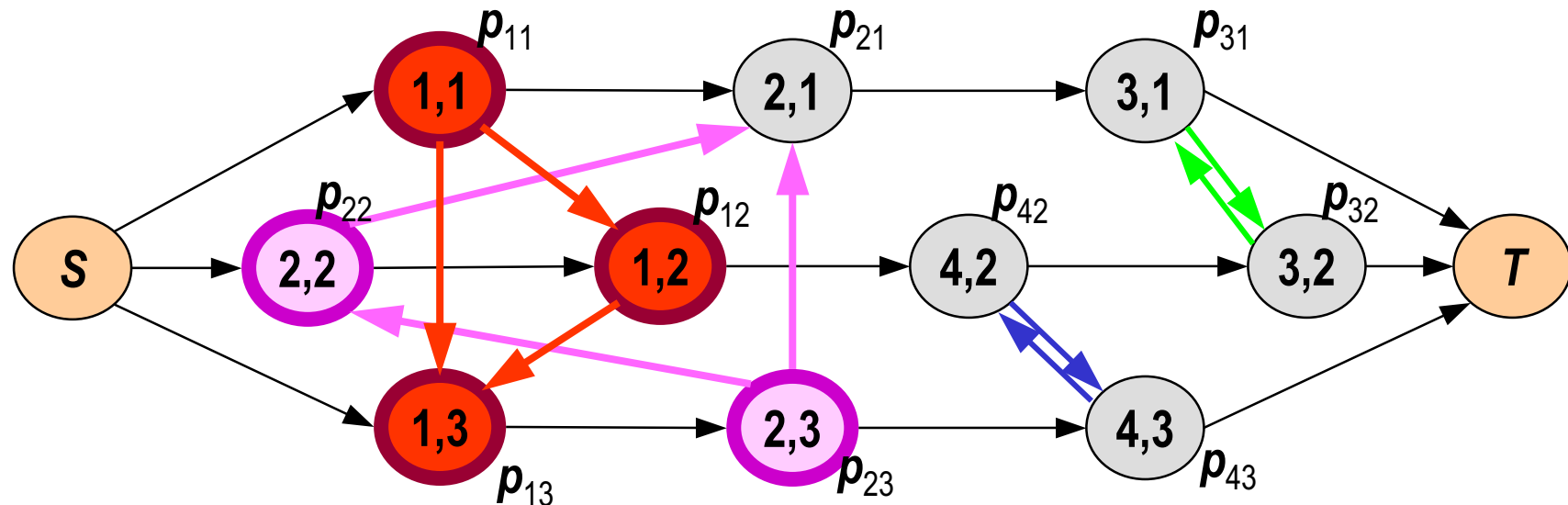
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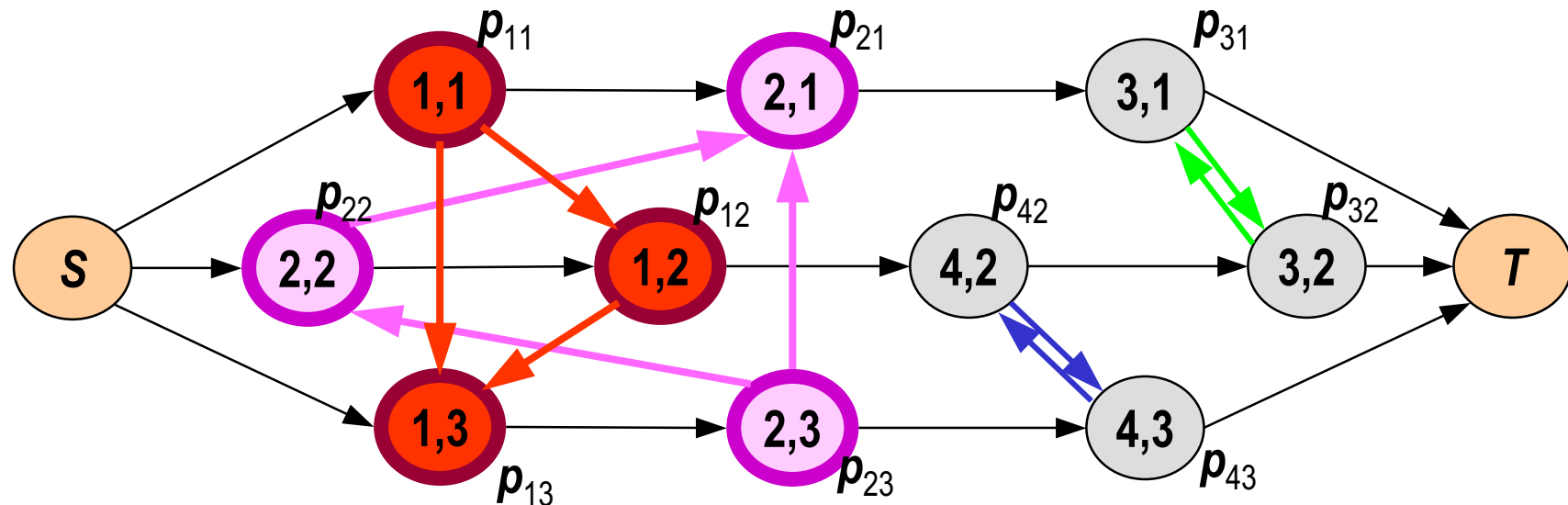
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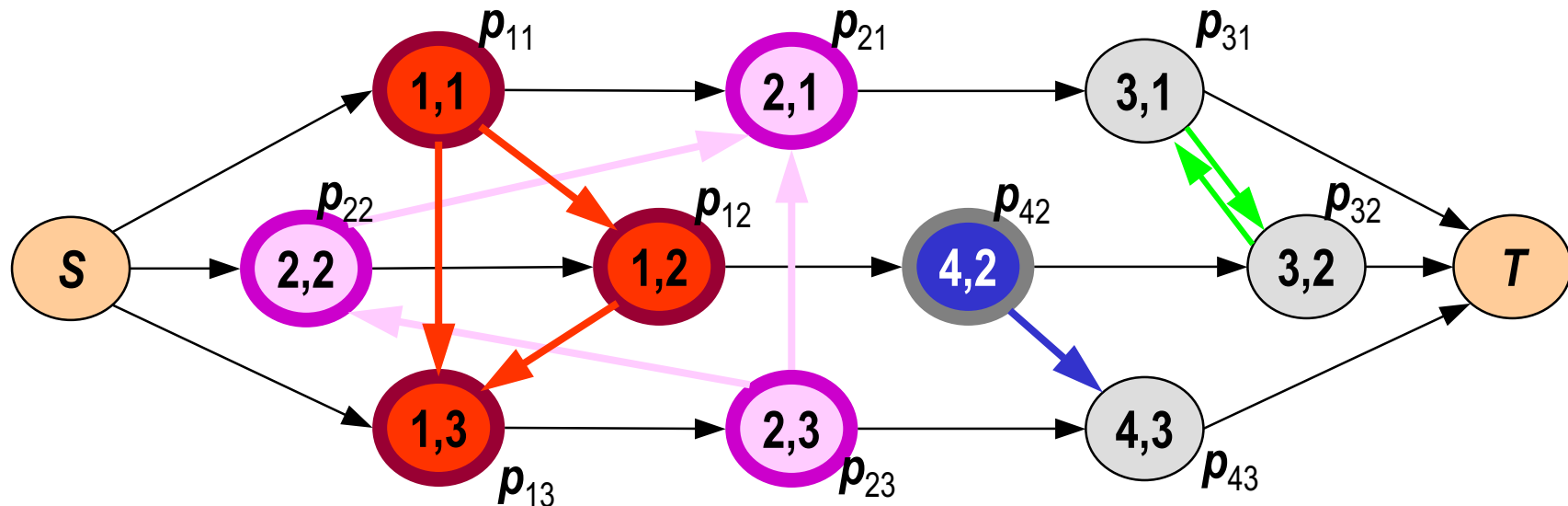
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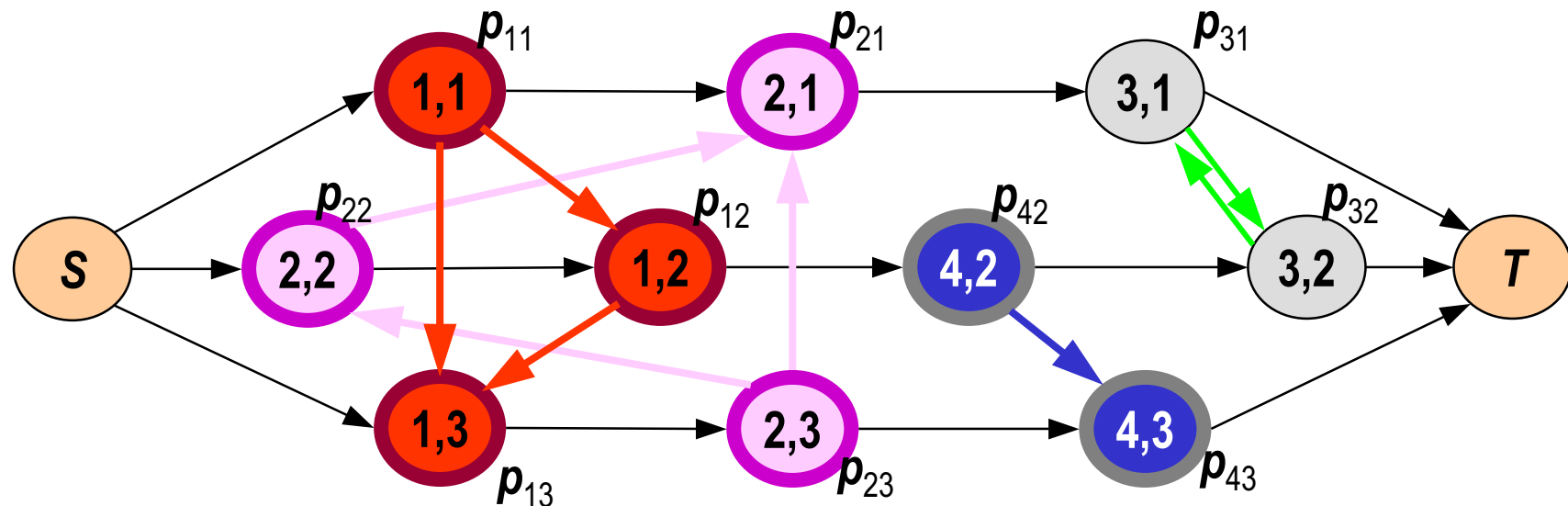
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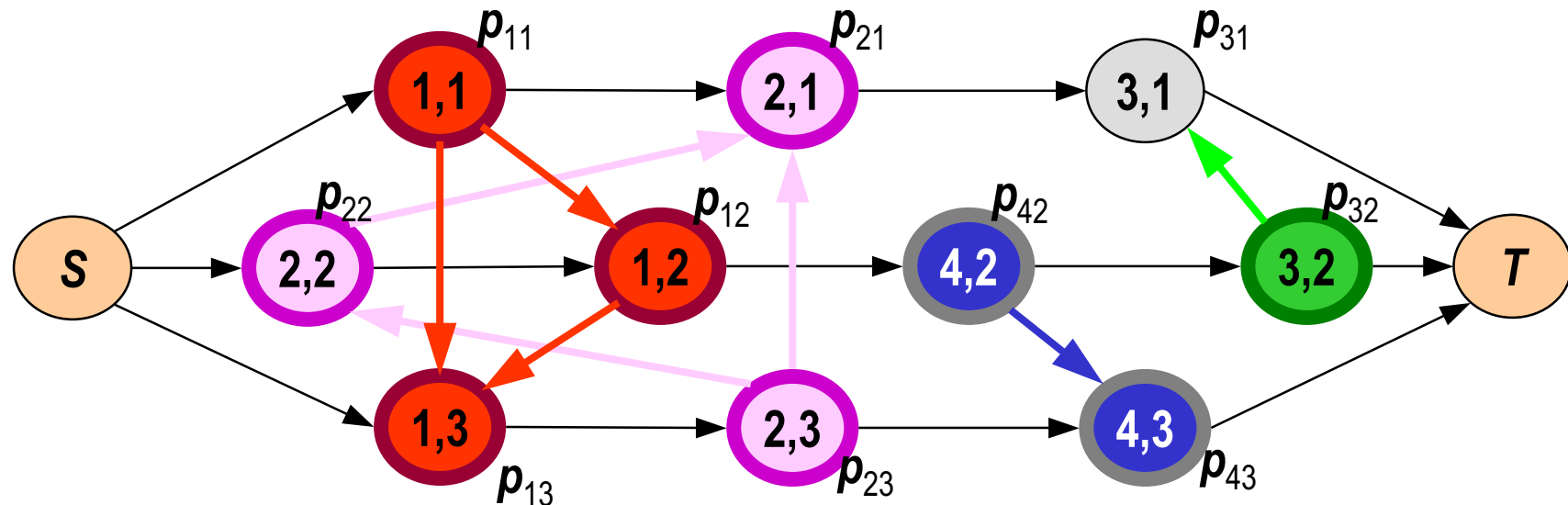
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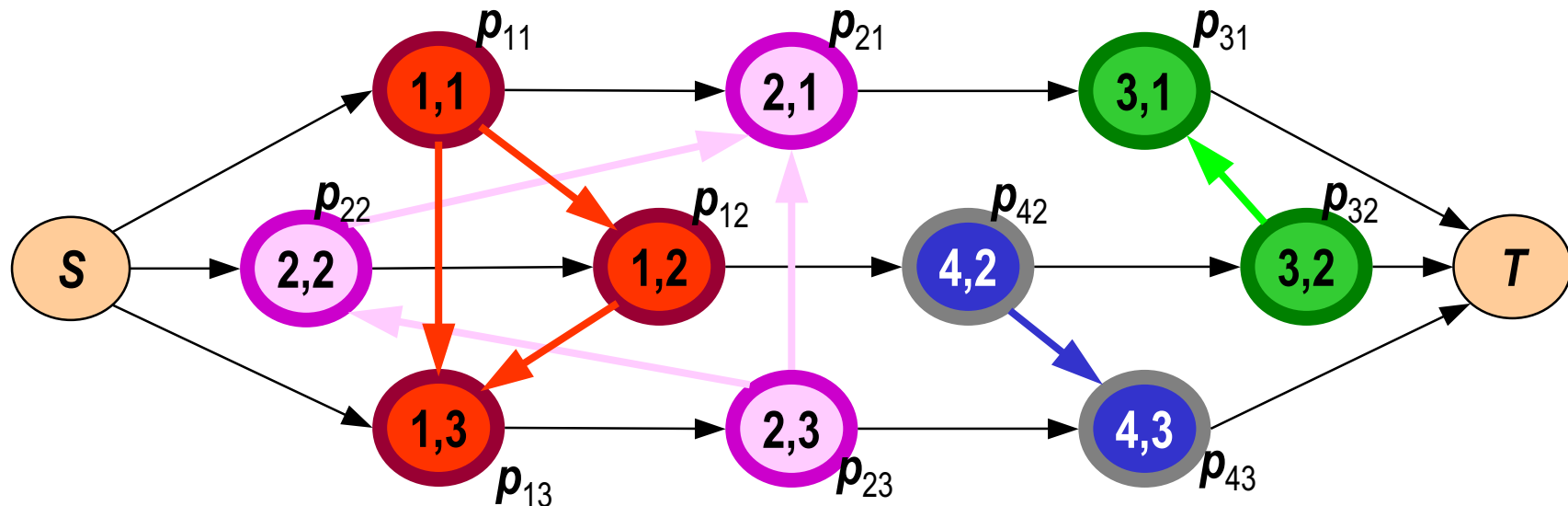
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Job Shop: Disjunctive Graph Model

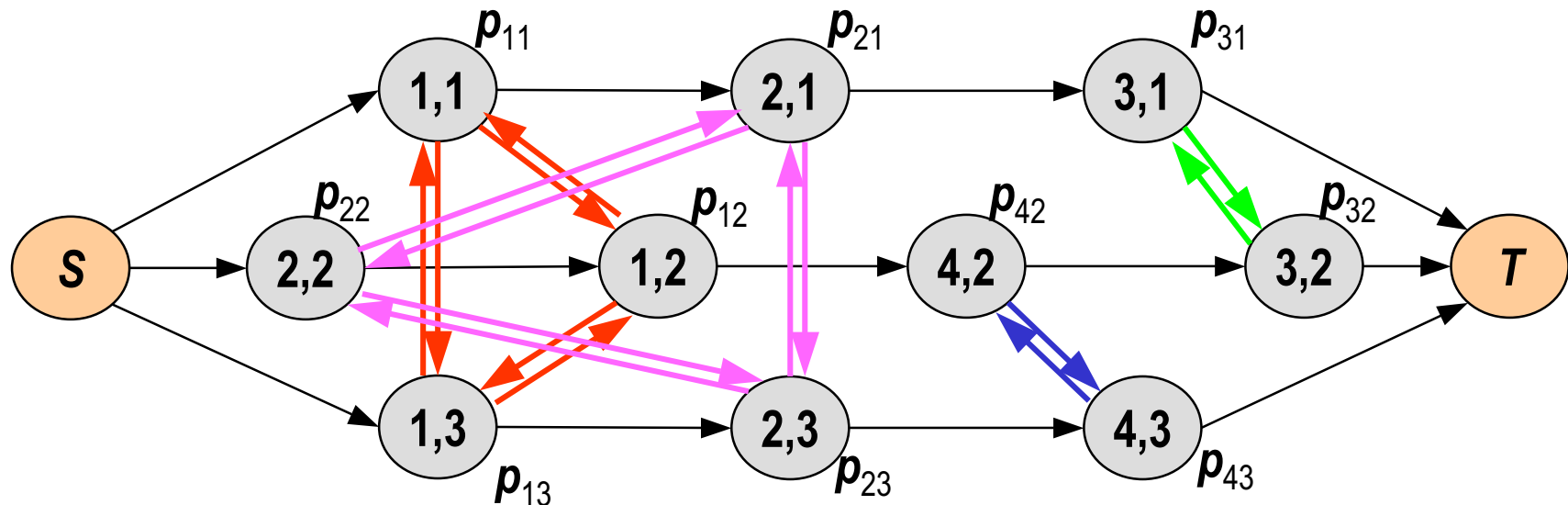
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Job Shop: Disjunctive Graph Model

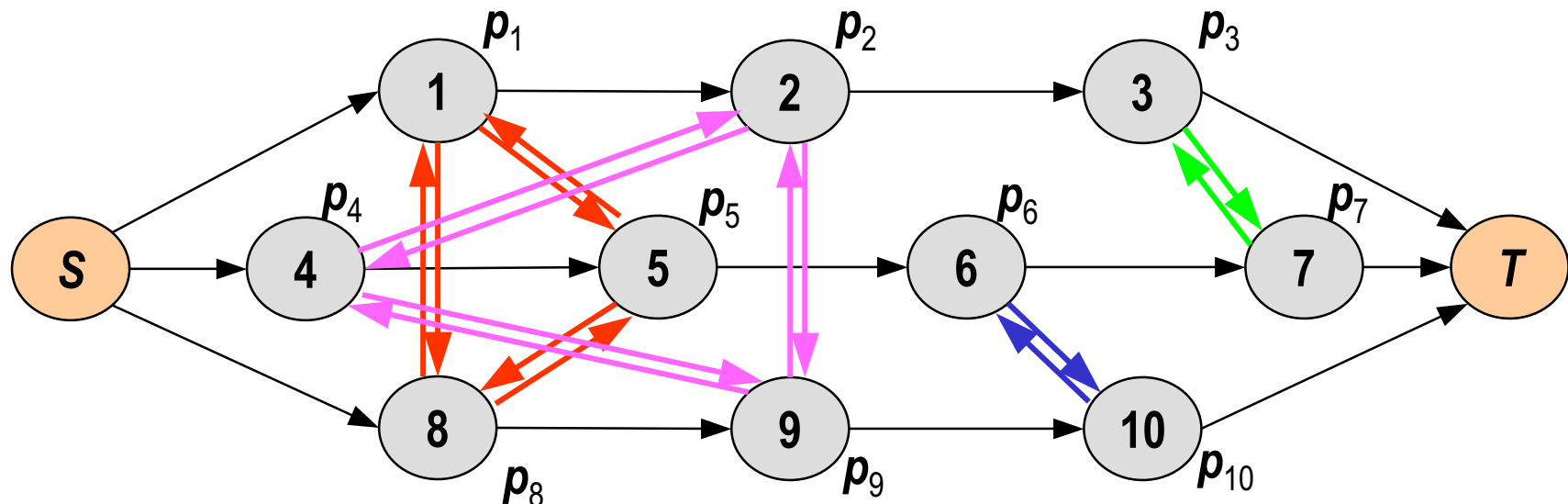
Disjunctive graph $G=(N, A, B)$:

- Nodes N correspond to all operations
- Conjunctive arcs A represent the precedence relations between operations of a single job
- Disjunctive arcs B link the operations processed by the same machine

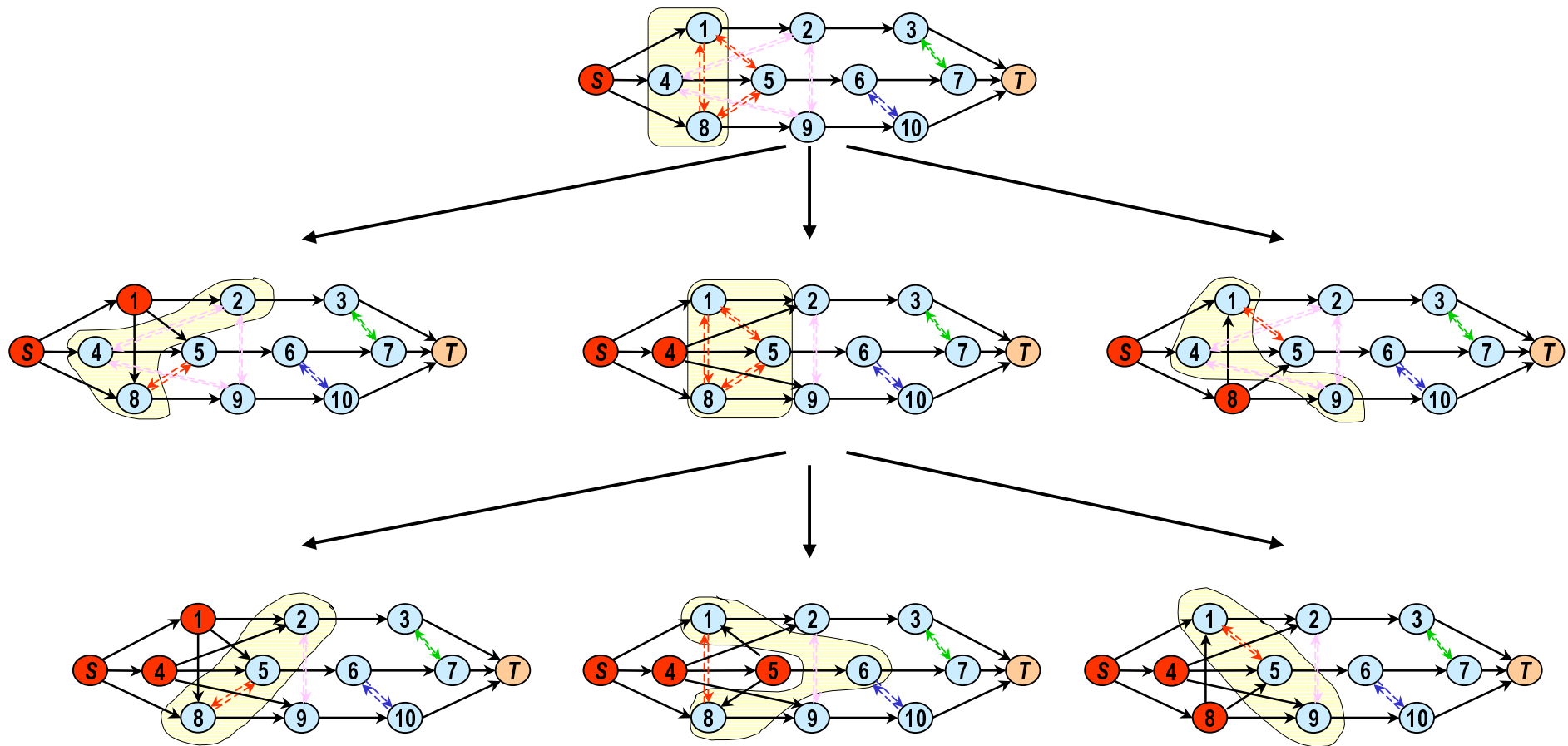


Job Shop: Branch & Bound

- Operations 1, 2, ..., q
- O denotes the set of “schedulable operations” (whose predecessors have already been scheduled)
- “**Scheduling**” of operation i means replacing each pair of disjunctive arcs incident to i by a conjunctive arc starting from i

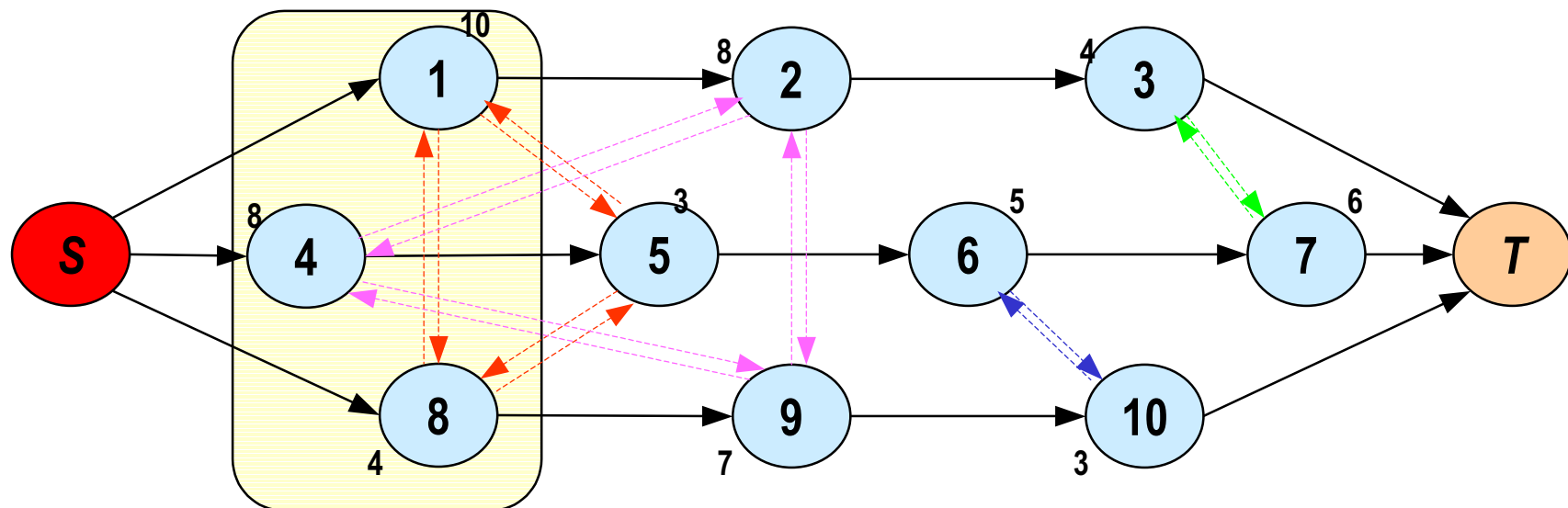


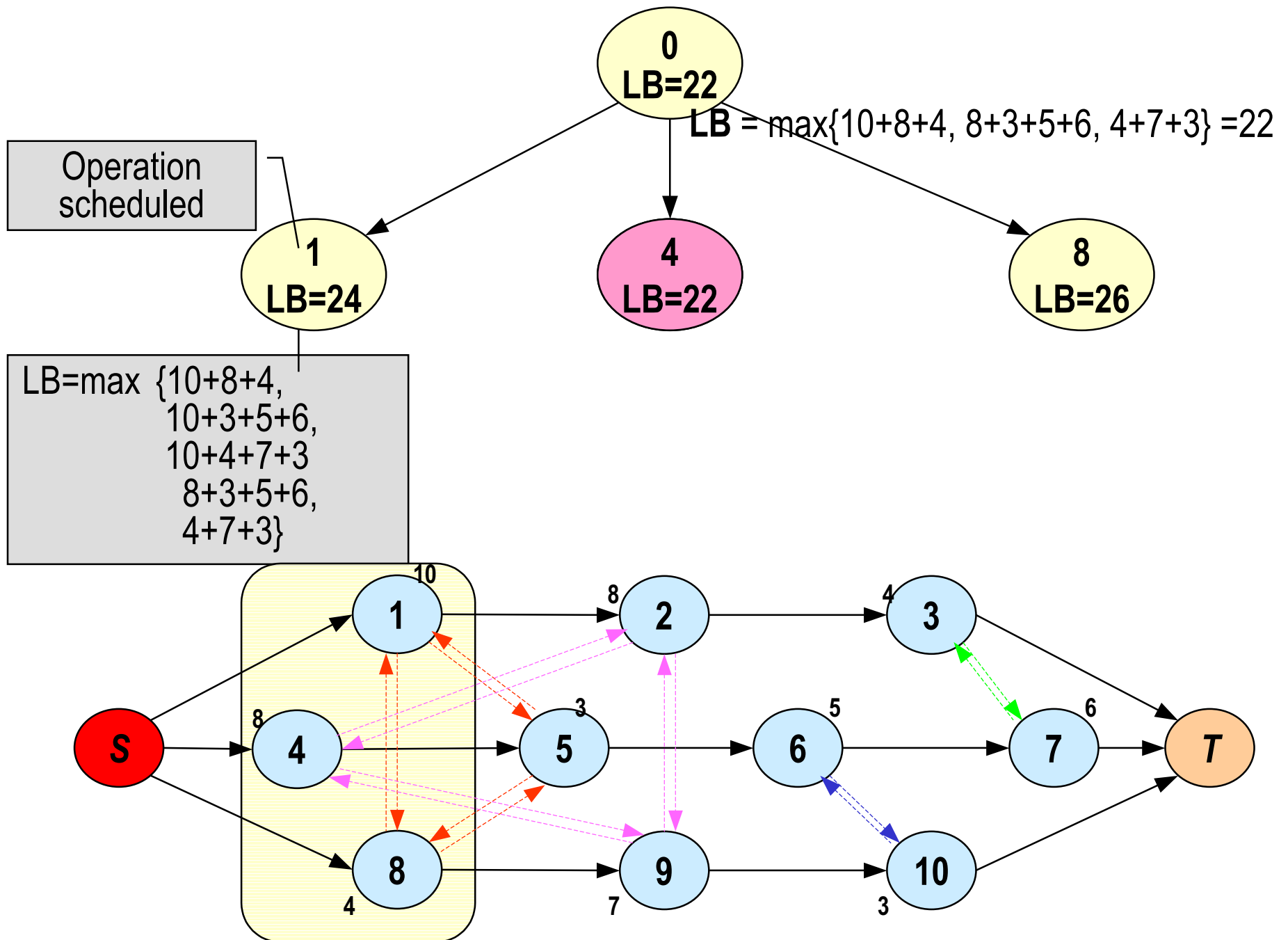
Job Shop: Branch & Bound

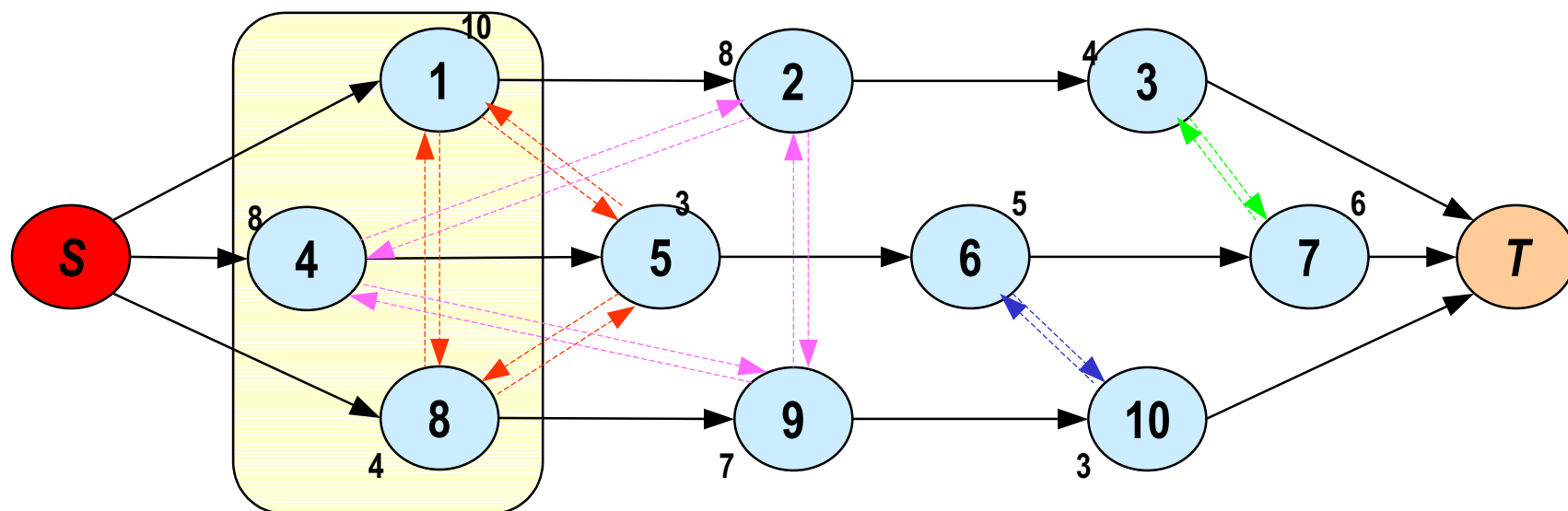
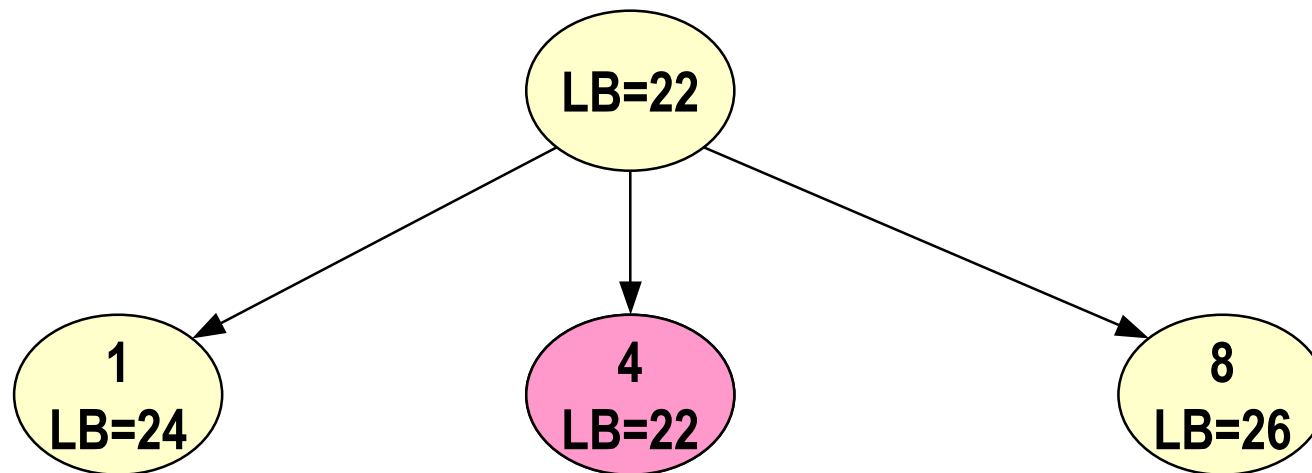


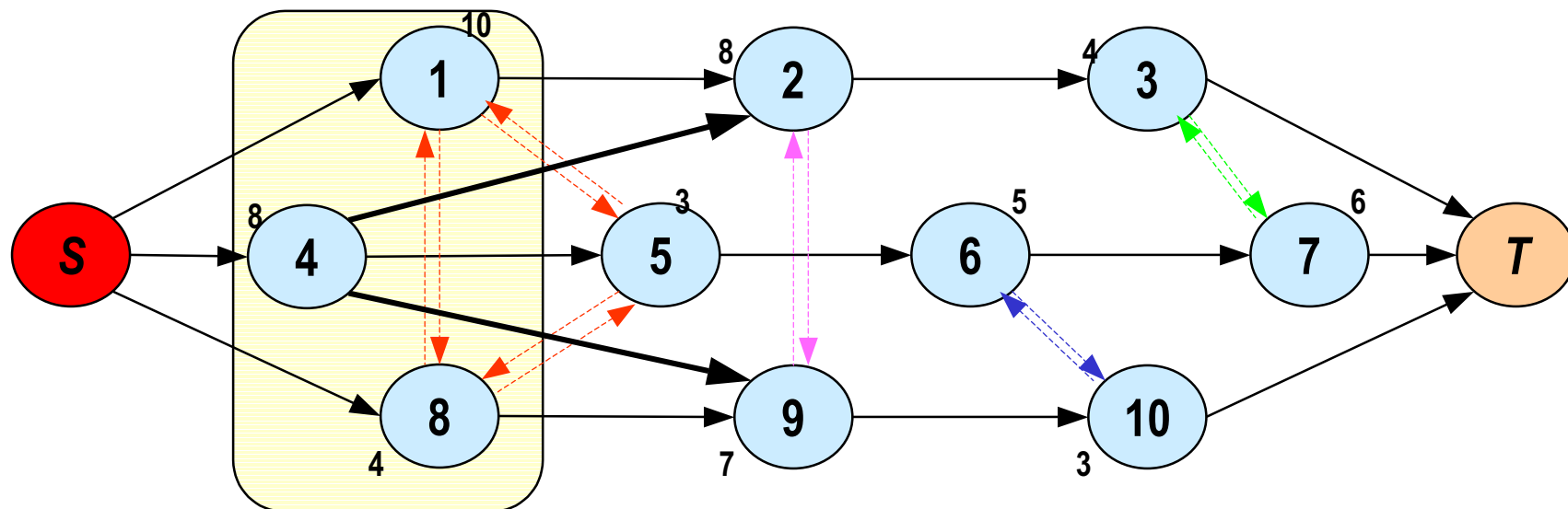
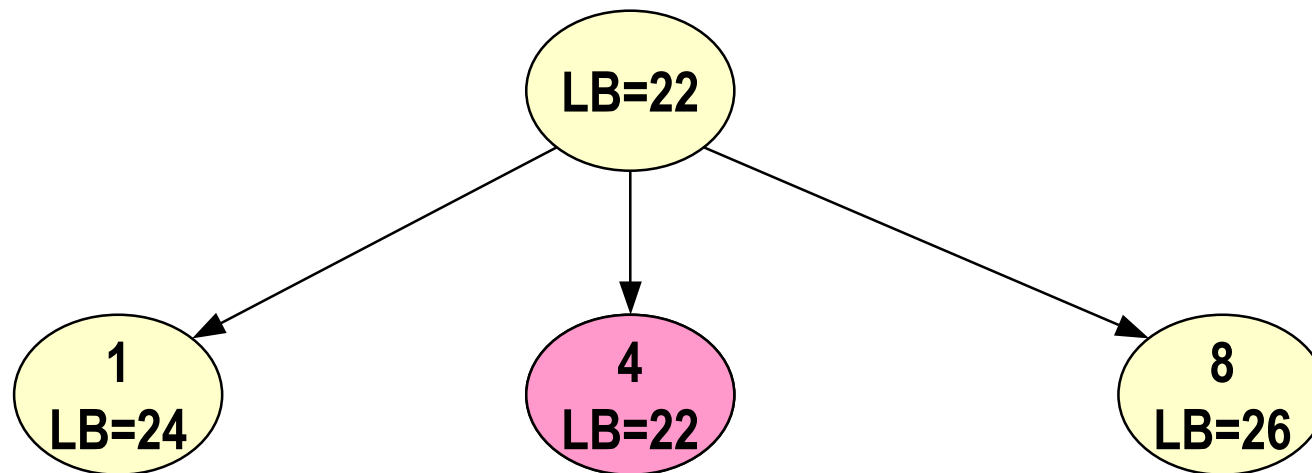
Job Shop: Branch & Bound

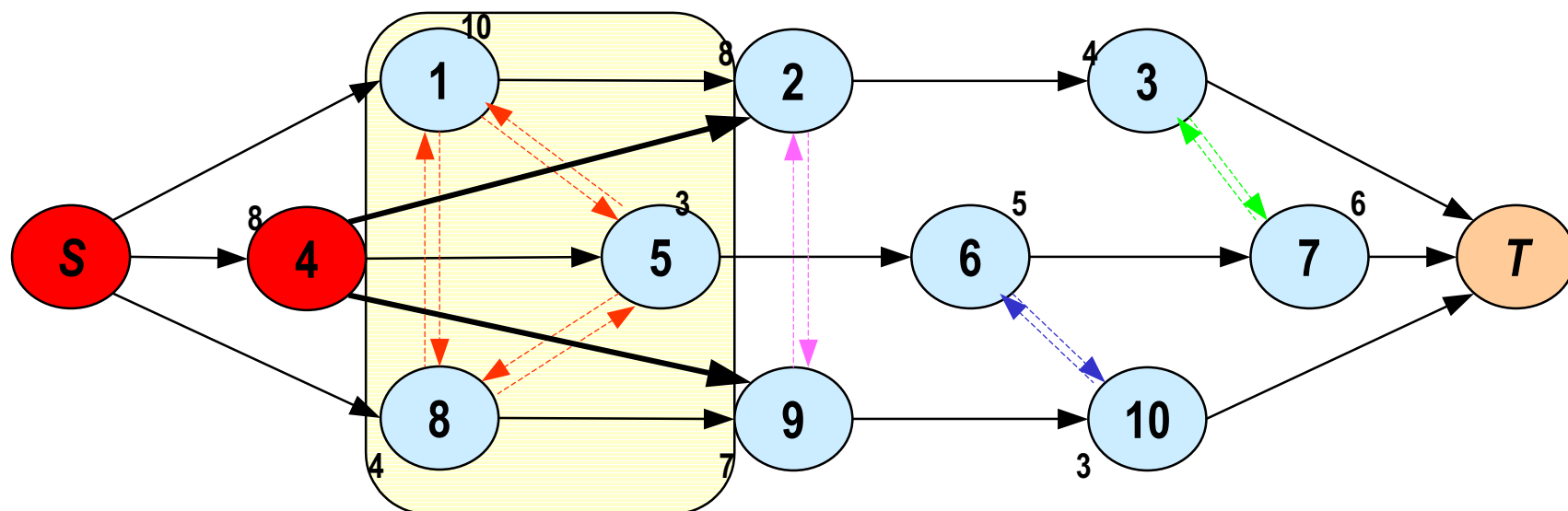
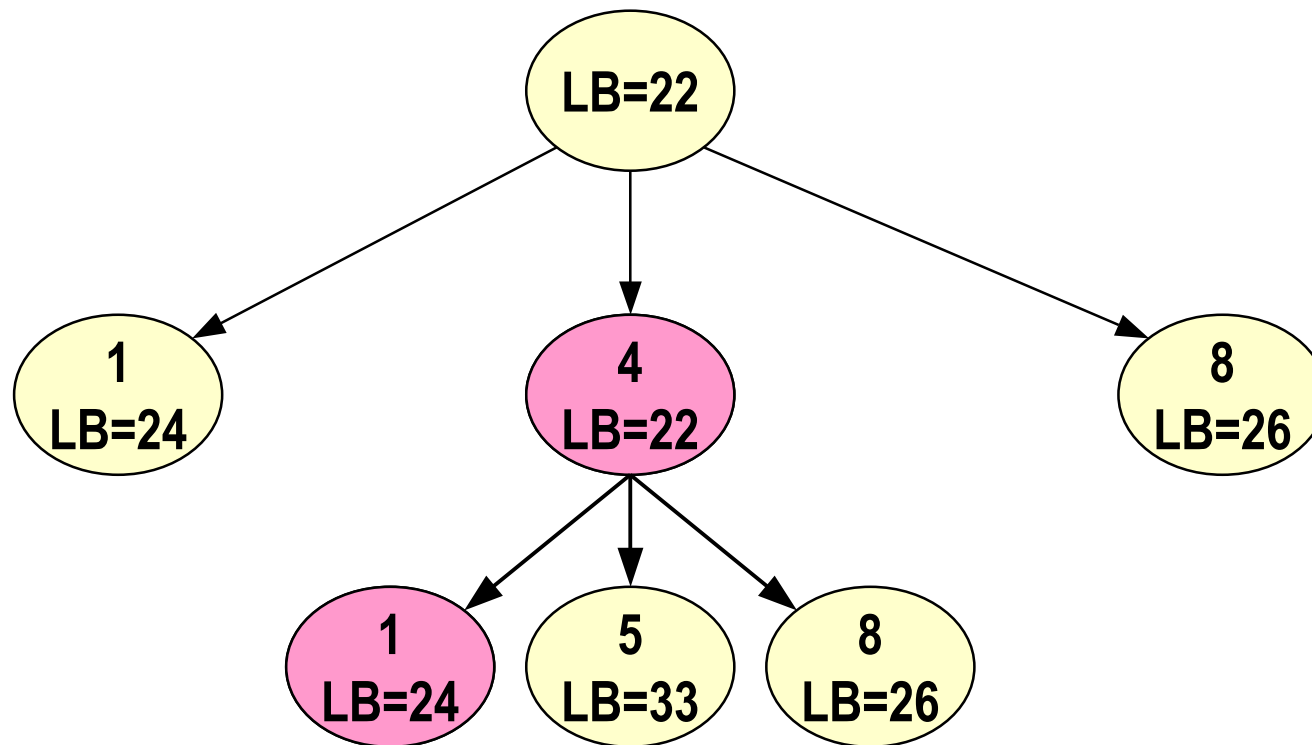
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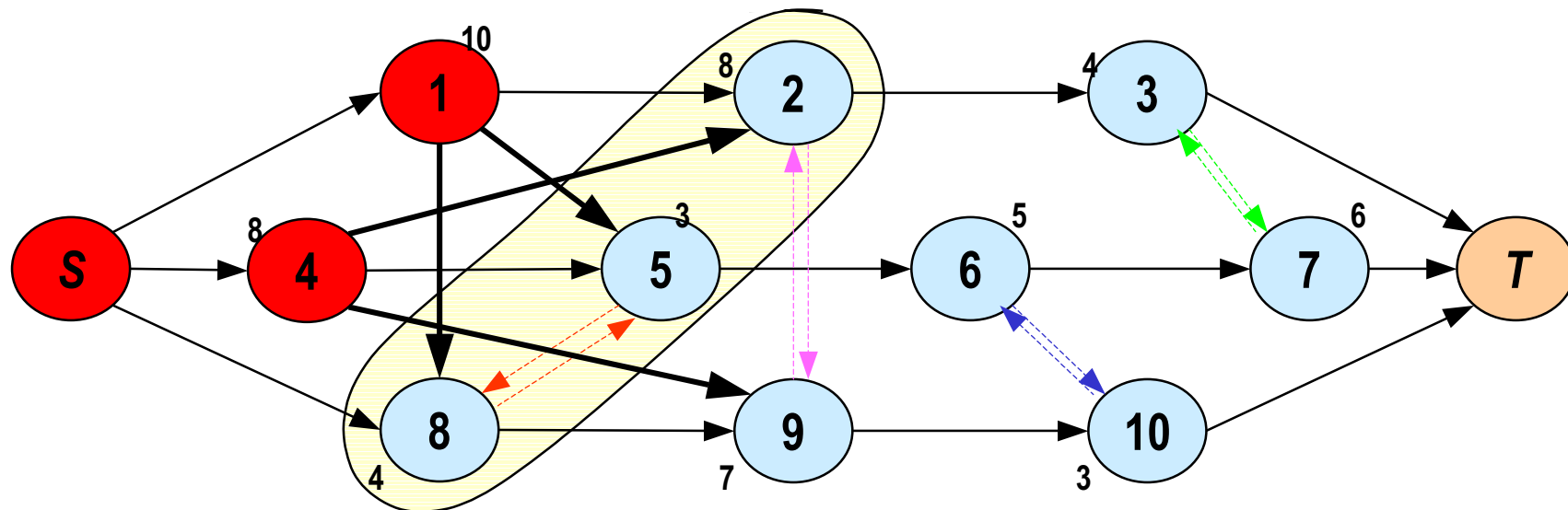
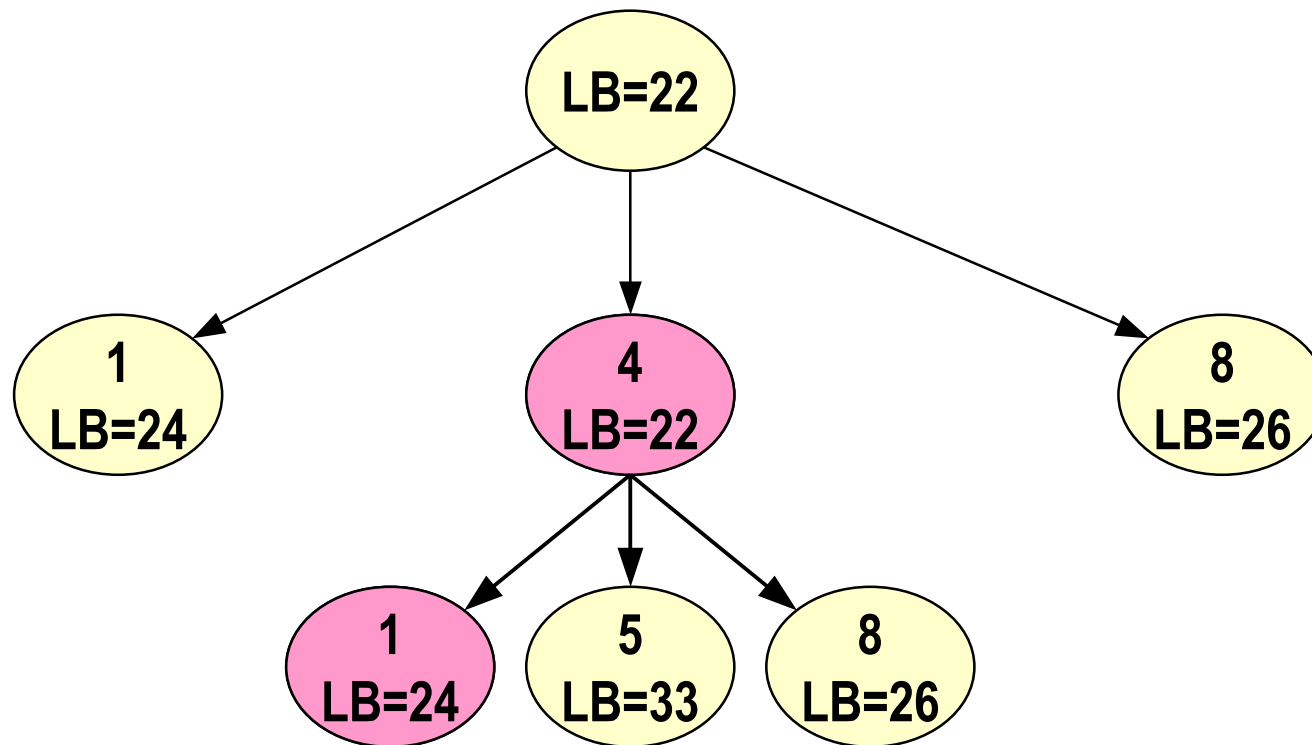


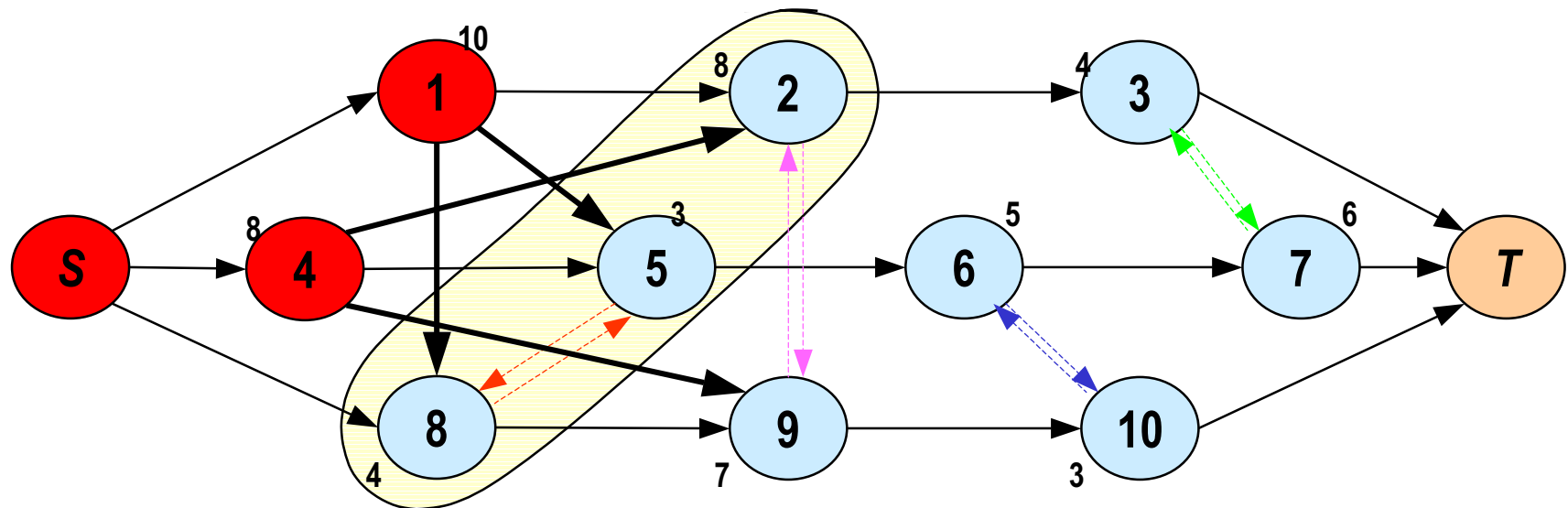
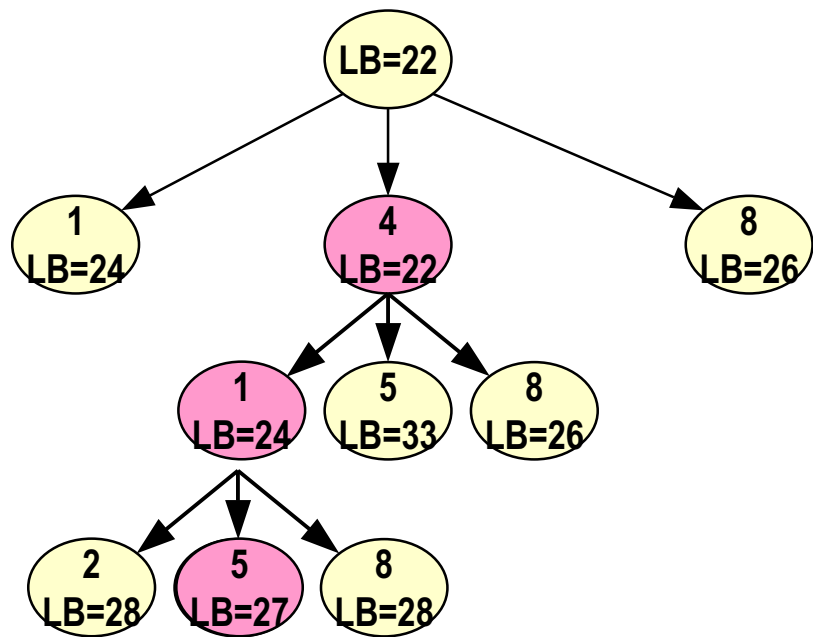


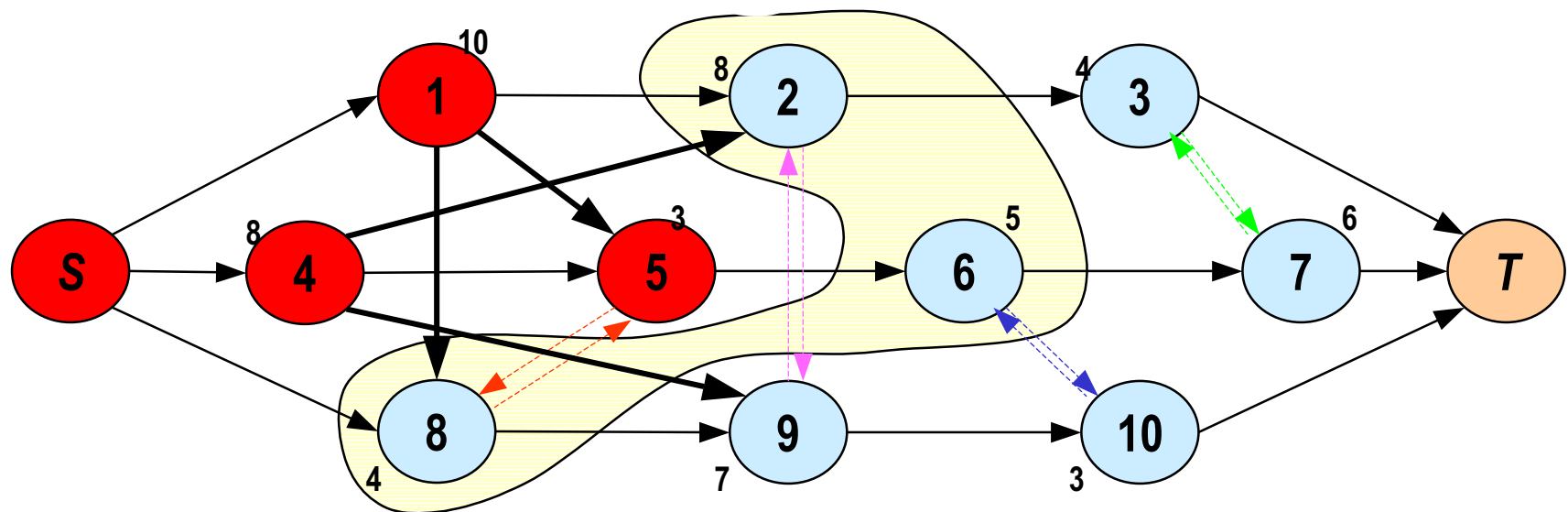
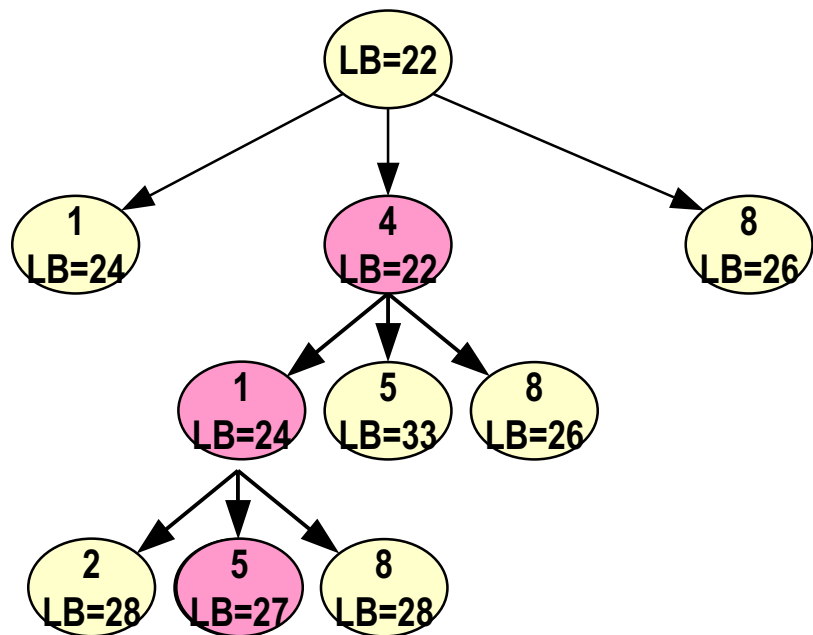


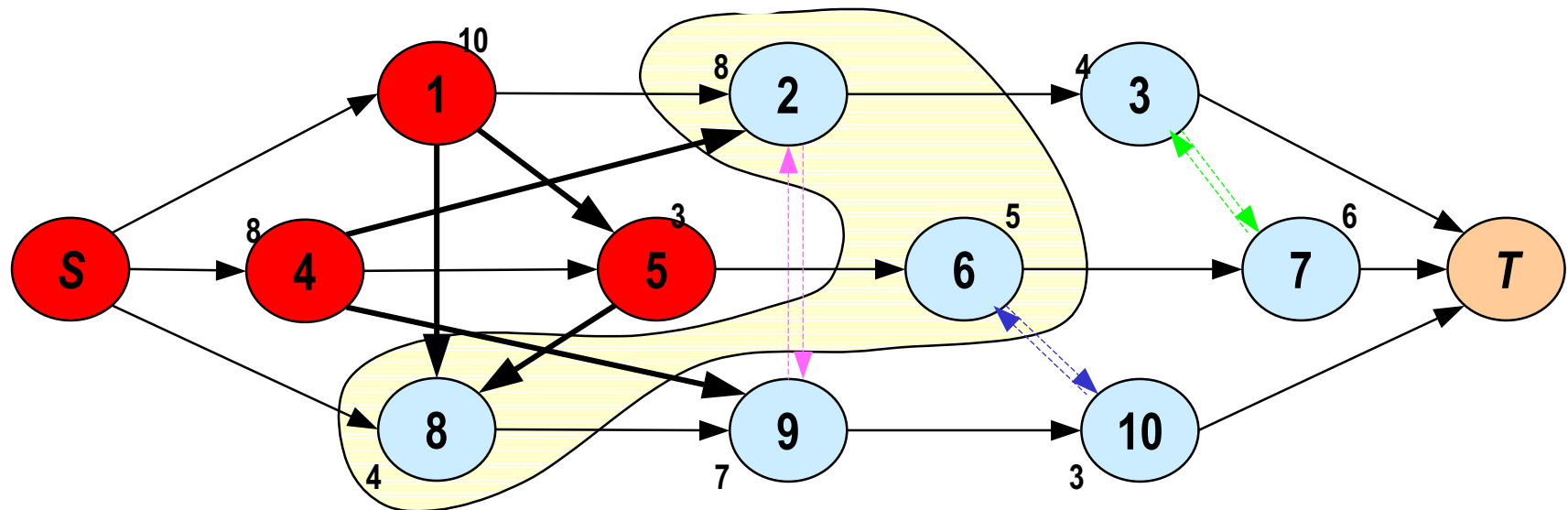
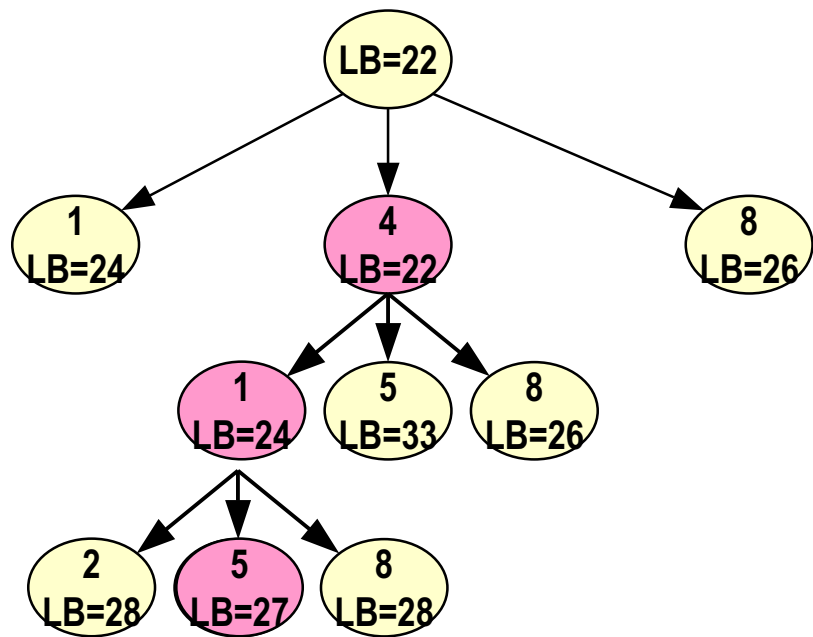


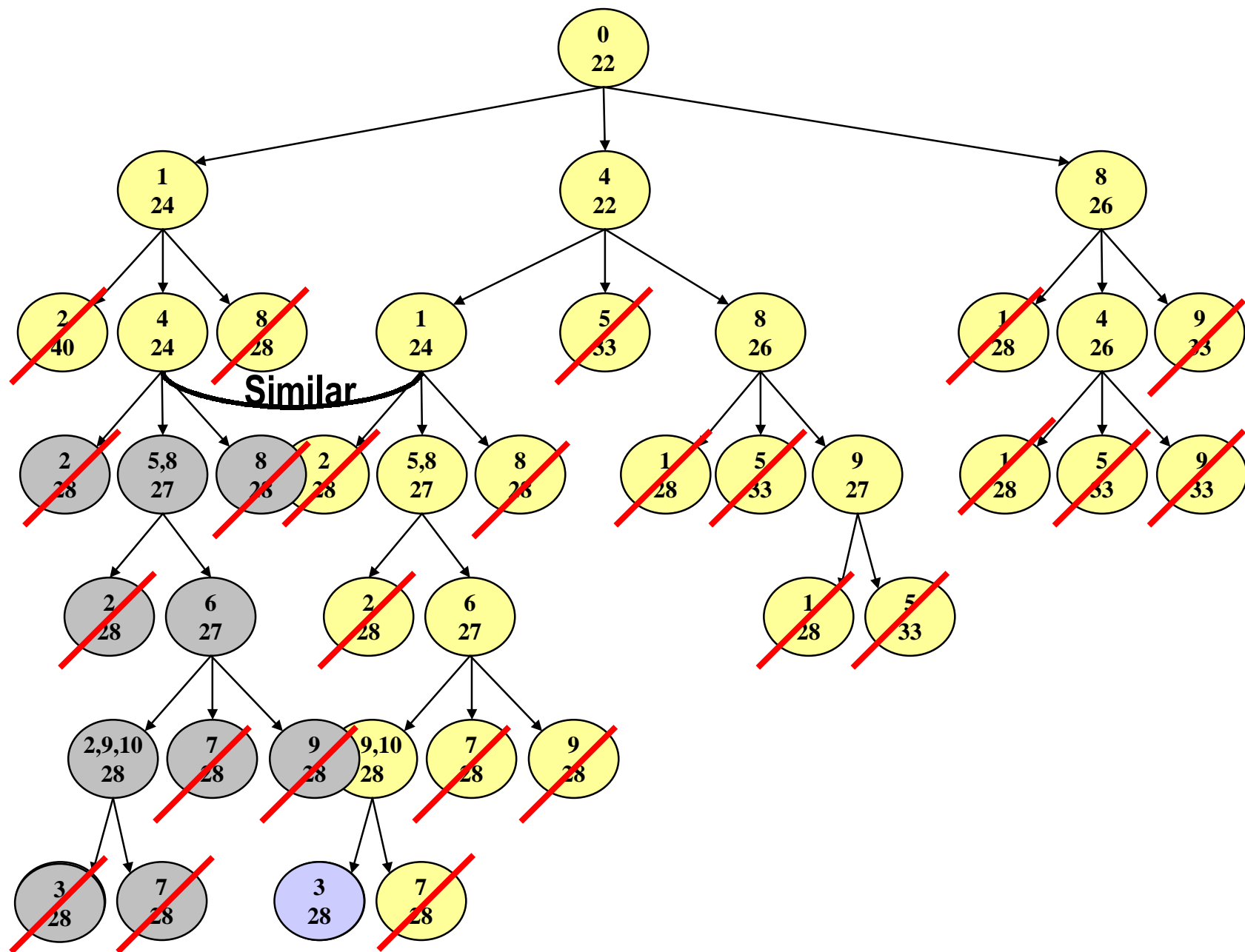












Job Shop: Branch & Bound

Advantages of Branch & Bound algorithm:

- Finds an optimal solution

Disadvantages of Branch & Bound algorithm:

- Extremely time-consuming: the number of nodes in a branching tree can be too large.

Algorithm can construct several feasible schedules and then develop “bad” branches of the tree. The number of the nodes grows exponentially without improving the best solution obtained.