Aircraft Routing and Scheduling

DETAILS EXAMPLE 11.3.1 IN TEXT

Initial master Problem:

T = 2

M1 = 2

M2 = 2

S1 =

1-1	2-1	3-1	4-1	5-1	6-1	7-1	8-1	9-1	10-1
0	0	1	1	0	0	0	0	0	0
0	0	0	0	0	0	0	1	0	1
0	0	0	1	0	0	0	0	0	0
0	0	1	0	0	0	0	1	0	0
0	1	1	1	0	0	0	0	0	0
0	0	0	0	1	0	0	1	0	1
0	1	1	0	1	0	1	0	0	0
0	0	0	1	0	0	0	0	0	1
0	0	0	0	0	1	0	1	0	1
0	1	0	0	1	1	1	0	1	0
0	0	0	0	1	1	1	0	1	0
0	1	0	0	0	1	1	0	1	0

Oih

0	1	1	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	1	0
0	0	0	0	0	1	0	1	0	1
0	0	0	0	1	0	1	0	0	0

Dih

0	1	1	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1
0	0	0	0	0	1	1	0	1	0
0	0	0	0	1	0	0	1	0	0

S2 =

1-2	2-2	3-2	4-2	5-2	6-2	7-2	8-2	9-2
0	1	0	1	0	0	0	0	0
0	0	0	0	0	0	1	0	1
0	0	0	1	0	0	0	1	0
0	1	0	0	0	0	1	0	0
0	1	0	0	0	0	0	0	0
0	0	1	1	0	0	1	0	1
0	1	1	0	0	1	0	0	0
0	0	0	1	0	0	0	0	1
0	0	0	0	1	0	1	0	1
0	0	1	0	1	1	0	1	0
0	0	1	0	1	1	0	1	0
0	0	0	0	1	1	0	1	0

Oih

0	1	0	1	0	0	0	0	0
0	0	0	0	0	0	0	1	0
0	0	0	0	1	0	1	0	1
0	0	1	0	0	1	0	0	0

Dih

0	1	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0	1
0	0	0	0	1	1	0	1	0
0	0	1	0	0	0	1	0	0

πij

$\pi 1$

1	2	3	4	5	6	7	8	9	10	11	12
450	300	500	400	900	900	900	900	1500	1500	1500	1500

$\pi 2$

1	2	3	4	5	6	7	8	9	10	11	12
450	450	500	500	1000	1000	1000	1000	1350	1350	1350	1350

πli

S1

1-1	2-1	3-1	4-1	5-1	6-1	7-1	8-1	9-1	10-1
-375	4800	2650	2750	4800	6000	5400	3100	4500	3600

S2

1-2	2-2	3-2	4-2	5-2	6-2	7-2	8-2	9-2
0	2950	4700	2950	5400	5050	3300	4550	3800

Solving the master problem through linear programming will result in the optimal solution of schedules 4-1, 7-1 and 8-1 from S1 with values of 2/3, and schedules 1-2 = 1 , 2-2, 8-2 and 9-2 from S2 with values of 1, 1/3, 1/3,and 1/3 with a resulting profit of 11,267. An optimal integer solution exists for this problem with schedules 1-1 and 4-1 from S1, and 6-2 and 7-2 from S2, with profit 10725.

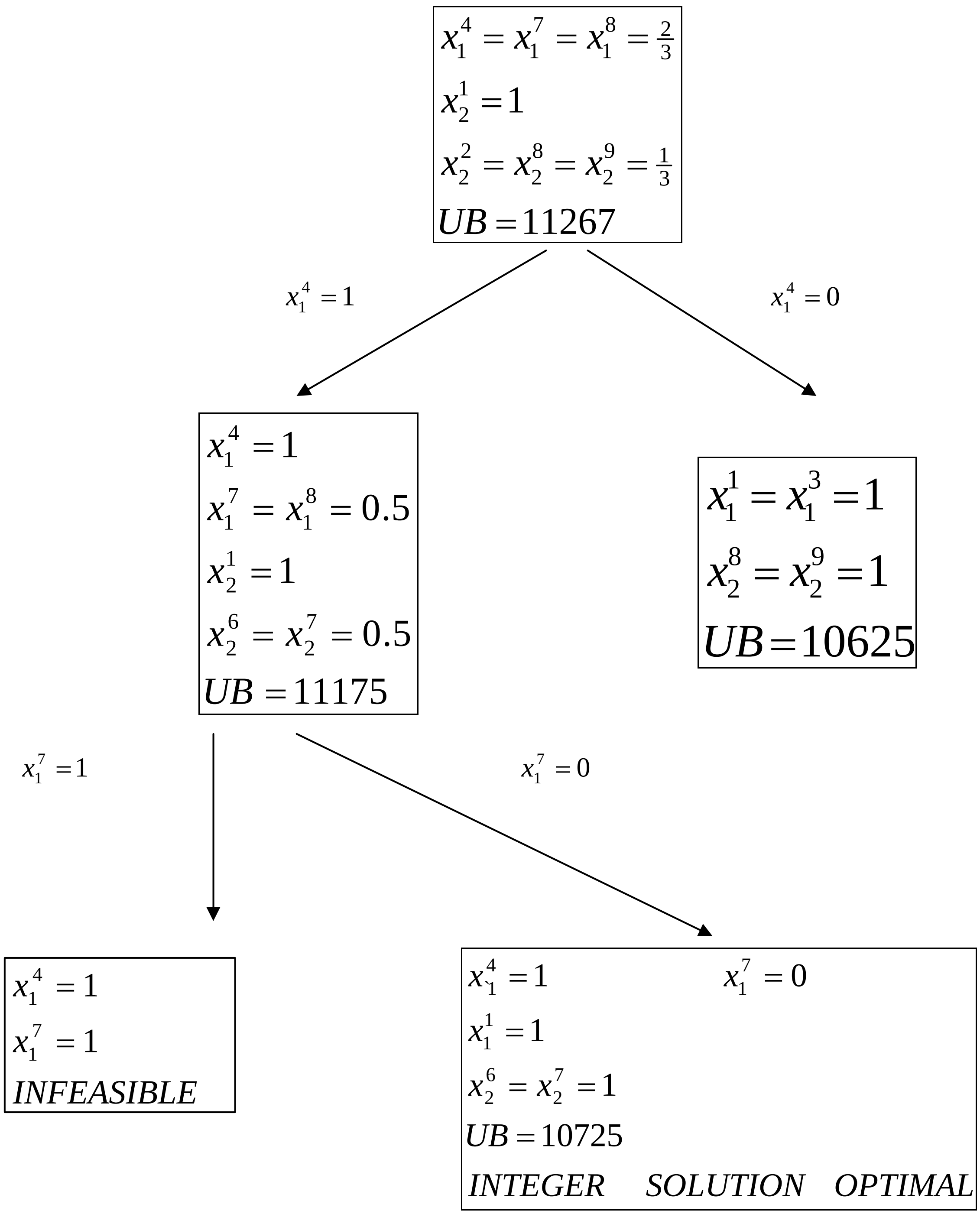

Figure 11.3 Branch-and-Bound Tree for Aircraft Scheduling Problem

Sub-problem

Dual Variables:

In this problem, the dual variables associated with the flight legs αj are as follows:

1	2	3	4	5	6	7	8	9	10	11	12
0	-616.66	216.66	2650	2550	2733.33	-2250	0	1500	4333.33	0	183.33

The dual variable β for type 1 is –16.66. The dual variable for type 2 aircraft is 0.

The dual variable for the origin and destination airports 0,0,0,3150 for large planes and 0,0,183.33,2966.66 for small planes for sfo, lax, nyc and sea.

Candidate schedules

Now that dual variables are known candidate schedules to be included in the master problem can be considered.

Potential schedules: (all turn times standard and included in flight leg duration)

Duration of flight legs:

1	2	3	4	5	7	12
1.5	1.5	1.5	1.5	3	3	6

Windows of flight legs (earliest departure, latest arrival)

1	2	3	4	5	7	12
0500,1200	1200,2359	0500,1200	1200,2359	0500,1200	0500,1200	1200,0500

1: The attached directed graph will show the following results for the following potential schedules:

Schedule A2: (450-0) + (500-216.67) + (900-2733.33) + (900-0) - -16.66 – 0 = -183.34

Schedule A3: (500-216.67) + (1500-4333.33) + (1500-0) + (1500-183.33) –16.66 – 0 = 283.33

Other schedules are checked for the same, and ones with positive results are included.

The following schedules are generated from the directed graph, ones with positive results included in the master problem.

	11-1	12-1	13-1	14-1	15-1
	0	1	0	1	1
	0	0	0	0	0
	1	1	0	1	1
	0	0	0	0	0
	0	0	0	0	0
	0	1	1	1	1
	0	0	1	1	1
	0	1	0	0	0
	0	0	0	0	0
	1	0	1	0	1
	1	0	1	0	0
	1	0	0	0	1
Profit	5000	2750	4800	2750	5750
Dual variable	4733.33	2933.34	4800	683.34	5200.00333
Potential profit	283.33	-183.33	0	2066.66	549.996667

Oih

0	0	0	0	0
1	0	0	0	0
0	0	1	0	0
0	1	0	1	1

Dih

0	0	0	0	0
0	0	0	0	0
1	0	1	0	0

0 1 0 1 1

2. Generating small plane schedules is as follows, calculated in the same manner as above:

	10-2	11-2	12-2	13-2	14-2	15-2
	0	1	0	1	1	1
	1	1	0	0	0	1
	0	1	0	1	1	1
	1	1	0	0	0	1
	1	1	0	0	0	0
	0	0	1	1	1	0
	1	0	1	1	1	0
	0	1	0	0	0	0
	0	0	0	0	0	0
	0	0	1	0	1	0
	0	0	1	0	0	0
	0	0	0	0	1	0
Profit	2950	3900	4700	2950	5650	1900
Dual variable	2233.33	4700	4816.67	700	5216.67	2250
Potential profit	716.667	-800	-116.67	2250	433.333	-350

Oih

1 0 0 0 0 1
0 0 0 0 0 0
0 1 1 0 0 0
0 0 0 1 1 0

Dih

1 0 0 0 0 1
0 0 0 0 0 0
0 1 1 0 0 0
0 0 0 1 1 0

The schedules with positive potential profit are included in the master problem.

Master Problem

The new master problem when solves through linear programming has the following schedules. Some of the schedule numbers have changed from the notation given in the potential schedules, ie 14-1 became 12-1 as potential schedules 12 and 13 were either negative or zero for the potential profit.

S1:

0	0	1	1	0	0	0	0	0	0	0	1	1
0	0	0	0	0	0	0	1	0	1	0	0	0
0	0	0	1	0	0	0	0	0	0	1	1	1
0	0	1	0	0	0	0	1	0	0	0	0	0
0	1	1	1	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	1	0	1	0	1	1
0	1	1	0	1	0	1	0	0	0	0	1	1
0	0	0	1	0	0	0	0	0	1	0	0	0
0	0	0	0	0	1	0	1	0	1	0	0	0
0	1	0	0	1	1	1	0	1	0	1	0	1
0	0	0	0	1	1	1	0	1	0	1	0	0
0	1	0	0	0	1	1	0	1	0	1	0	1

S2:

0	1	0	1	0	0	0	0	0	0	1	1
0	0	0	0	0	0	1	0	1	1	0	0
0	0	0	1	0	0	0	1	0	0	1	1
0	1	0	0	0	0	1	0	0	1	0	0
0	1	0	0	0	0	0	0	0	1	0	0
0	0	1	1	0	0	1	0	1	0	1	1
0	1	1	0	0	1	0	0	0	1	1	1
0	0	0	1	0	0	0	0	1	0	0	0
0	0	0	0	1	0	1	0	1	0	0	0
0	0	1	0	1	1	0	1	0	0	0	1
0	0	1	0	1	1	0	1	0	0	0	0
0	0	0	0	1	1	0	1	0	0	0	1

πij

$\pi 1$

1	2	3	4	5	6	7	8	9	10	11	12
450	300	500	400	900	900	900	900	1500	1500	1500	1500

π2

1	2	3	4	5	6	7	8	9	10	11	12
450	450	500	500	1000	1000	1000	1000	1350	1350	1350	1350

πli

1

1-1	2-1	3-1	4-1	5-1	6-1	7-1	8-1	9-1	10-1	11-1	12-1	13-1
-375	4800	2650	2750	4800	6000	5400	3100	4500	3600	5000	2750	5750

2

1-2	2-2	3-2	4-2	5-2	6-2	7-2	8-2	9-2	10-2	13-2	14-2
0	2950	4700	2950	5400	5050	3300	4550	3800	2950	2950	5650

When the new master problem is solved through linear programming the result is:

X4-1,6-1,7-1,8-1,4-2,11-2 = 0.5 and X2-1 = 1

The branch and bound tree for the master problem is as follows:

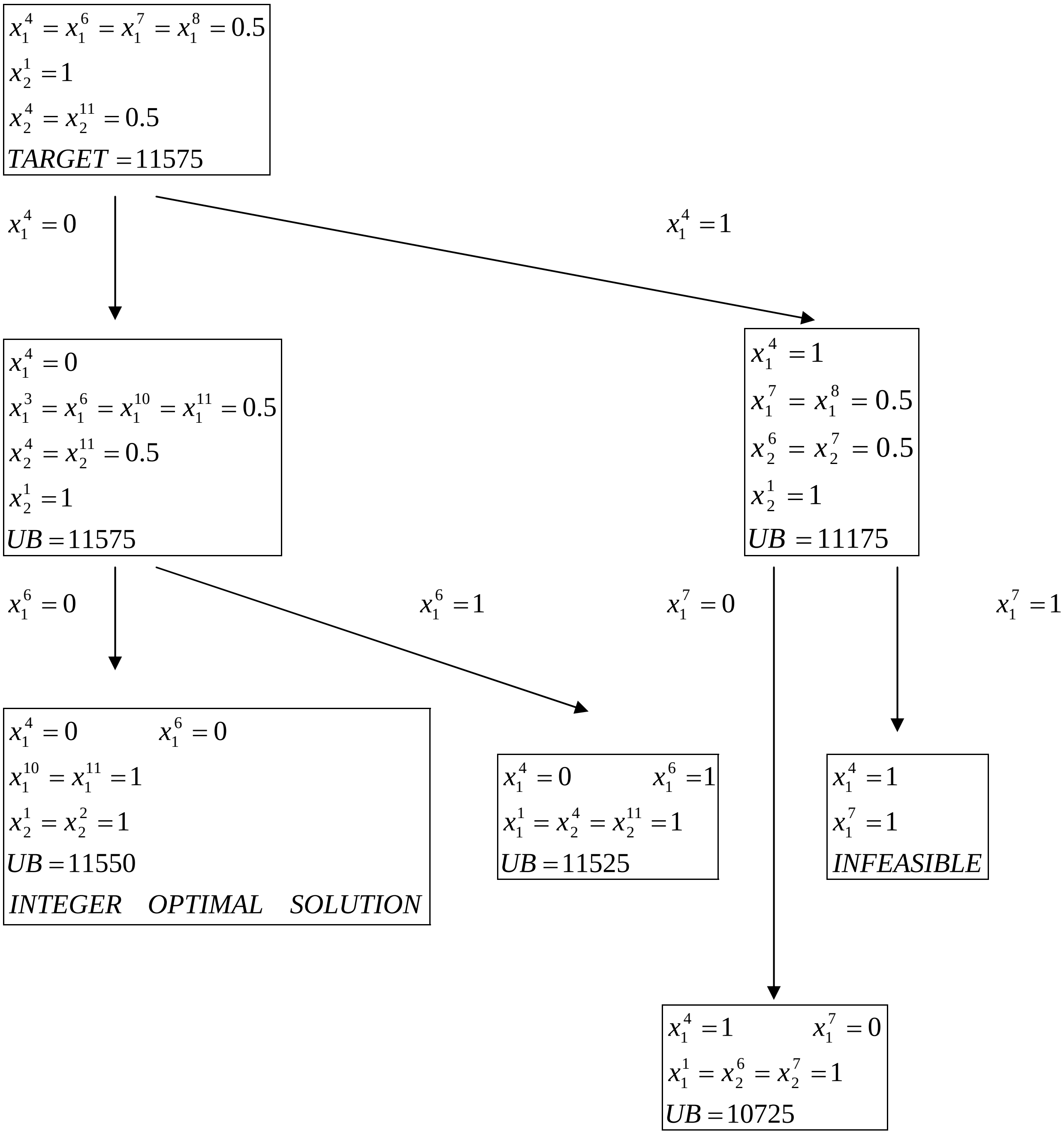

Figure 11.5   Branch-and-Bound Tree for Aircraft Scheduling Problem