

## Addendum to Coordination and Flexibility in Supply Contracts with Options

### B Notation

$c_i$	unit wholesale price for firm orders to be delivered in period $i$ .
$c_e$	unit exercise price.
$c_o$	unit option price.
$c_L$	unit labor cost of production.
$c_r$	unit cost of raw material.
$\gamma$	additional unit labor cost of production in the expedited mode.
$X_L$	production quantity of the supplier in the cheaper mode of production; that is at labor cost of $c_L$ per unit.
$v_r^s$	unit salvage value of raw material for the supplier.
$v_f^s$	unit salvage value of finished goods for the supplier.
$h^s$	unit period one holding cost for finished goods at the supplier.
$Q_i$	order quantity to be delivered at the beginning of period $i$ , $i \in \{1, 2\}$ at a wholesale price of $c_i$ .
$M$	number of options purchased at the beginning of period one at a unit price of $c_o$ .
$m$	number of options exercised at the beginning of period two at a price of $c_e$ .
$r$	unit selling price of finished good to the consumer.
$h_i^b$	unit holding cost of the buyer in period $i \in \{1, 2\}$ .
$p_i$	unit shortage penalty cost incurred by the buyer in period $i \in \{1, 2\}$ .
$\tilde{v}_f^b$	unit salvage value of finished goods for the buyer.
$v_f^b$	effective salvage value of finished goods for the buyer; equal to $\tilde{v}_f^b - h_2^b$ .
$D_i$	demand in period $i \in \{1, 2\}$ assumed to be normally distributed.
$\mu_i$	mean of $D_i$ .
$\sigma_i$	standard deviation of $D_i$ .
$\rho$	correlation coefficient of $D_1$ and $D_2$ .
$F_{D_i}(\cdot)$	conditional distribution function of $D_i$ .
$f_{D_i}(\cdot)$	conditional density function of $D_i$ .
$\Phi(\cdot)$	distribution function of standard normal.
$c_b$	price the supplier gives the buyer for goods returned at the end of the season.
$t_{bs}$	cost the supplier pays to ship goods back from the buyer at the end of the season.

## C Expressions for the Remaining Profit Functions

- **Profit function of the system in CS,  $v_f^s \leq v_f^b$  and  $v_r^s + c_L(1 + \gamma) \leq v_f^b$ :**

$$\Pi_{CS}(X_1^c, X_L^c, X_3^c) = J_1^c(X_1^c) + J_L^c(X_L^c) + J_3^c(X_3^c),$$

where  $X_L^c = X_3^c$ ,  $J_1^c(X_1^c)$  is as in (11) and

$$J_3^c(X_3^c) = (p_2 + r - c_r - c_L - h^s)X_3^c - (p_2 + r - v_f^b) \int_0^{X_3^c} (X_3^c - d_{12}) dF_{D_1+D_2}(d_{12}) \quad (14)$$

- **Profit function of the system in CS,  $v_f^s > v_f^b$  and  $v_r^s + c_L(1 + \gamma) \leq v_f^s$ :**

$$\Pi_{CS}(X_1^c, X_L^c, X_3^c) = J_1^c(X_1^c) + J_3^c(X_3^c),$$

where  $X_L^c = X_3^c$  and

$$\begin{aligned} J_1^c(X_1^c) = & -p_2\mu_2 - \mu_1(p_1 + p_2) - (p_1 + h_1^b) \int_0^{X_1^c} (X_1^c - d_1) dF_{D_1}(d_1) \\ & + (p_1 + p_2 + r - v_f^s + h^s)X_1^c + (p_2 + r - v_f^s) \int_{\frac{X_1^c - \delta_1^c}{\eta}}^{\infty} (\delta_1^c + \eta d_1 - X_1^c) dF_{D_1}(d_1) \\ & - (p_2 + r - v_f^b) \left[ \int_0^{\frac{X_1^c - \delta_1^c}{\eta}} \int_0^{X_1^c - d_1} (X_1^c - d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \right. \\ & \left. + \int_{\frac{X_1^c - \delta_1^c}{\eta}}^{\infty} \int_0^{\delta_1^c + (\eta-1)d_1} (\delta_1^c + (\eta-1)d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \right], \end{aligned} \quad (15)$$

$$\begin{aligned} J_3^c(X_3^c) = & (v_f^s - c_r - c_L - h^s)X_3^c - (p_2 + r - v_f^s) \int_{\frac{X_3^c - \delta_3^c}{\eta}}^{\infty} (\delta_1^c + \eta d_1 - X_3^c) dF_{D_1}(d_1) \\ & - (p_2 + r - v_f^b) \left[ \int_{\frac{X_3^c - \delta_3^c}{\eta}}^{\infty} \int_0^{X_3^c - d_1} (X_3^c - d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \right. \\ & \left. - \int_{\frac{X_3^c - \delta_3^c}{\eta}}^{\infty} \int_0^{\delta_1^c + (\eta-1)d_1} (\delta_1^c + (\eta-1)d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \right]. \end{aligned} \quad (16)$$

- **Profit function of the system in CS,  $v_f^s > v_f^b$  and  $v_r^s + c_L(1 + \gamma) > v_f^s$ :**

$$\Pi_{CS}(X_1^c, X_L^c, X_3^c) = J_1^c(X_1^c) + J_L^c(X_L^c) + J_3^c(X_3^c),$$

where  $J_1^c(X_1^c)$  is as in (15),  $J_3^c(X_3^c)$  is as in (13), and

$$\begin{aligned} J_L^c(X_L^c) = & (v_f^s - v_r^s - c_L - h^s)X_L^c - (p_2 + r - v_f^s) \int_{\frac{X_L^c - \delta_1^c}{\eta}}^{\infty} (\delta_1^c + \eta d_1 - X_L^c) dF_{D_1}(d_1) \\ & + (p_2 + r - c_L(1 + \gamma) - v_r^s) \int_{\frac{X_L^c - \delta_2^c}{\eta}}^{\infty} (\delta_2^c + \eta d_1 - X_L^c) dF_{D_1}(d_1) \\ & - (p_2 + r - v_f^b) \left[ \int_{\frac{X_L^c - \delta_1^c}{\eta}}^{\frac{X_L^c - \delta_2^c}{\eta}} \int_0^{X_L^c - d_1} (X_L^c - d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \right. \\ & \left. + \int_{\frac{X_L^c - \delta_2^c}{\eta}}^{\infty} \int_0^{X_L^c - d_1} (X_L^c - d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \right] \end{aligned}$$

$$\begin{aligned}
& - \int_{\frac{X_1^c - \delta_1^c}{\eta}}^{\infty} \int_0^{\delta_1^c + (\eta-1)d_1} (\delta_1^c + (\eta-1)d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \\
& + \int_{\frac{X_2^c - \delta_2^c}{\eta}}^{\infty} \int_0^{\delta_2^c + (\eta-1)d_1} (\delta_2^c + (\eta-1)d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \Big]. \tag{17}
\end{aligned}$$

## D Proofs

### Derivation of (7)

Let us begin by simplifying (6). Collecting terms and taking expectations gives

$$\begin{aligned}
\Pi_1^b(Q_1, Q_2, M) &= E_{D_1} [rD_1 - r(D_1 - Q_1)^+ - c_o M - c_1 Q_1 - c_2 Q_2 - h_1^b (Q_1 - d_1)^+ - p_1 (d_1 - Q_1)^+ \\
&\quad + E_{D_2} [\Pi_2^b(m^* | d_1)]] \\
&= -c_o M - c_1 Q_1 - c_2 Q_2 - (h_1^b + p_1) \int_0^{Q_1} (Q_1 - d_1) dF_{D_1}(d_1) - p_1 (\mu_1 - Q_1) \\
&\quad + E_{D_1} [\Pi_2^b(m^* | d_1) + rD_1 - r(D_1 - Q_1)^+]. \tag{18}
\end{aligned}$$

Expanding out  $E_{D_1} [\Pi_2^b(m^* | d_1) + rD_1 - r(D_1 - Q_1)^+]$ , we get

$$\begin{aligned}
& E_{D_1} [\Pi_2^b(m^* | d_1) + rD_1 - r(D_1 - Q_1)^+] \\
&= \int_0^{\infty} \left[ (r - c_e) m^* - p_2 (\mu_2 - (\eta - 1) \mu_1 + \eta D_1 - Q_1 - Q_2 - m^*) + r(Q_1 + Q_2) \right. \\
&\quad \left. - (p_2 + r - v_f^b) \int_0^{Q_1 + Q_2 - d_1 + m^*} (Q_1 + Q_2 + m^* - d_1 - d_2) dF_{D_2|d_1}(d_2) \right] dF_{D_1}(d_1) \\
&= (p_2 + r - c_e) \int_0^{\infty} m^* dF_{D_1}(d_1) - p_2 (\mu_1 + \mu_2 - Q_1 - Q_2) + r(Q_1 + Q_2) \\
&\quad - (p_2 + r - v_f^b) \left( \int_0^{\frac{Q_1 + Q_2 - \delta^d}{\eta}} \int_0^{Q_1 + Q_2 - d_1} (Q_1 + Q_2 - d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \right. \\
&\quad + \int_{\frac{Q_1 + Q_2 - \delta^d}{\eta}}^{\frac{Q_1 + Q_2 + M - \delta^d}{\eta}} \int_0^{\delta^d + (\eta-1)d_1} (\delta^d + (\eta-1)d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \\
&\quad \left. + \int_{\frac{Q_1 + Q_2 + M - \delta^d}{\eta}}^{\infty} \int_0^{Q_1 + Q_2 + M - d_1} (Q_1 + Q_2 + M - d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \right). \tag{19}
\end{aligned}$$

Substituting,  $(X_1, X_2, X_3) = (Q_1, Q_1 + Q_2, Q_1 + Q_2 + M)$ , we rewrite  $\Pi^b(Q_1, Q_2, M)$  as

$$\begin{aligned}
\Pi_{DS}^b(X_1, X_2, X_3) &= -c_o X_3 + (r - (c_2 - c_o)) X_2 - (c_2 - c_1) X_1 - (h_1^b + p_1) \int_0^{X_1} (X_1 - d_1) dF_{D_1}(d_1) \\
&\quad - p_1 (\mu_1 - X_1) + E_{D_1} [\Pi_2^b((\delta^d + \eta D_1 - X_2)^+ - (\delta^d + \eta D_1 - X_3)^+)] \\
&\quad + r E_{D_1} [(\delta^d + \eta D_1 - X_2)^+ - (\delta^d + \eta D_1 - X_3)^+ - (X_2 + m^* - D_1)^+]. \tag{20}
\end{aligned}$$

where

$$E_{D_1} [\Pi_2^b((\delta^d + \eta D_1 - X_2)^+ - (\delta^d + \eta D_1 - X_3)^+) + r m^* - r(X_2 + m^* - D_1)^+]$$

$$\begin{aligned}
&= -p_2(\mu_1 + \mu_2 - X_2) + (p_2 + r - c_e) \int_{\frac{X_2 - \delta^d}{\eta}}^{\infty} (\delta^d + \eta d_1 - X_2) dF_{D_1}(d_1) \\
&\quad - (p_2 + r - c_e) \int_{\frac{X_3 - \delta^d}{\eta}}^{\infty} (\delta^d + \eta d_1 - X_3) dF_{D_1}(d_1) \\
&\quad - (p_2 + r - v_f^b) \int_0^{\frac{X_2 - \delta^d}{\eta}} \int_0^{X_2 - d_1} (X_2 - d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \\
&\quad - (p_2 + r - v_f^b) \int_{\frac{X_2 - \delta^d}{\eta}}^{\frac{X_3 - \delta^d}{\eta}} \int_0^{\delta^d + (\eta-1)d_1} (\delta^d + (\eta-1)d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \\
&\quad - (p_2 + r - v_f^b) \int_{\frac{X_3 - \delta^d}{\eta}}^{\infty} \int_0^{X_3 - d_1} (X_3 - d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1). \tag{21}
\end{aligned}$$

Finally, collecting terms and restating, we get our result. ■

**Proof of concavity of  $\Pi_{DS}^b(X_1, X_2, X_3)$ :** We begin by taking the first partials of  $J_1(X_1)$ ,  $J_2(X_2)$ , and  $J_3(X_3)$  (given in Appendix A) with respect to  $X_1$ ,  $X_2$  and  $X_3$ .

$$\begin{aligned}
\frac{\partial J_1(X_1)}{\partial X_1} &= -(h_1^b + p_1)F_{D_1}(X_1) + p_1 + c_2 - c_1 \\
\frac{\partial J_2(X_2)}{\partial X_2} &= p_2 + r - (c_2 - c_o) - (p_2 + r - c_e) \int_{\frac{X_2 - \delta^d}{\eta}}^{\infty} dF_{D_1}(d_1) \\
&\quad - (p_2 + r - v_f^b) \int_0^{\frac{X_2 - \delta^d}{\eta}} \int_0^{X_2 - d_1} dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \\
&\quad + \frac{p_2 + r - v_f^b}{\eta} \left( \int_0^{X_2 - \frac{X_2 - \delta^d}{\eta}} \left( X_2 - \frac{X_2 - \delta^d}{\eta} - d_2 \right) dF_{D_2|\frac{X_2 - \delta^d}{\eta}}(d_2) f_{D_1} \left( \frac{X_2 - \delta^d}{\eta} \right) \right. \\
&\quad \left. - \int_0^{\delta^d + (\eta-1)\frac{X_2 - \delta^d}{\eta}} \left( \delta^d + (\eta-1)\frac{X_2 - \delta^d}{\eta} - d_2 \right) dF_{D_2|\frac{X_2 - \delta^d}{\eta}}(d_2) f_{D_1} \left( \frac{X_2 - \delta^d}{\eta} \right) \right) \\
&= c_o + c_e - c_2 + (p_2 + r - c_e)F_{D_1} \left( \frac{X_2 - \delta^d}{\eta} \right) \\
&\quad - (p_2 + r - v_f^b) \int_0^{\frac{X_2 - \delta^d}{\eta}} F_{D_2|d_1}(X_2 - d_1) dF_{D_1}(d_1) \\
\frac{\partial J_3(X_3)}{\partial X_3} &= -c_o + (p_2 + r - c_e) \int_{\frac{X_3 - \delta^d}{\eta}}^{\infty} dF_{D_1}(d_1) \\
&\quad - \frac{p_2 + r - v_f^b}{\eta} \int_0^{\delta^d + (\eta-1)\frac{X_3 - \delta^d}{\eta}} \left( \delta^d + (\eta-1)\frac{X_3 - \delta^d}{\eta} - d_2 \right) dF_{D_2|\frac{X_3 - \delta^d}{\eta}}(d_2) f_{D_1} \left( \frac{X_3 - \delta^d}{\eta} \right) \\
&\quad + \frac{p_2 + r - v_f^b}{\eta} \int_0^{X_3 - \frac{X_3 - \delta^d}{\eta}} \left( X_3 - \frac{X_3 - \delta^d}{\eta} - d_2 \right) dF_{D_2|\frac{X_3 - \delta^d}{\eta}}(d_2) f_{D_1} \left( \frac{X_3 - \delta^d}{\eta} \right) \\
&\quad - (p_2 + r - v_f^b) \int_{\frac{X_3 - \delta^d}{\eta}}^{\infty} \int_0^{X_3 - d_1} dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \\
&= p_2 + r - c_o - c_e - (p_2 + r - c_e)F_{D_1} \left( \frac{X_3 - \delta^d}{\eta} \right) \\
&\quad - (p_2 + r - v_f^b) \int_{\frac{X_3 - \delta^d}{\eta}}^{\infty} F_{D_2|d_1}(X_3 - d_1) dF_{D_1}(d_1)
\end{aligned}$$

The function  $\Pi_{DS}^b$  will be strictly concave if the Hessian matrix of  $-J_1(X_1) - J_2(X_2) - J_3(X_3)$  is positive definite. Due to the earlier transformations, all cross-partials will be zero. We need only to calculate the three second partials. For the derivations listed below, the following identity is needed.

$$F_{D_2|\frac{X_2-\delta^d}{\eta}} \left( X_2 - \frac{X_2 - \delta^d}{\eta} \right) = \Phi \left( \frac{X_2 - \frac{X_2 - \delta^d}{\eta} - \mu_2 - \rho \frac{\sigma_2}{\sigma_1} \left( \frac{X_2 - \delta^d}{\eta} - \mu_1 \right)}{\sigma_2 \sqrt{1 - \rho^2}} \right) = \Phi(k_m) \quad (22)$$

Using this the second partials can be calculated as follows:

$$\frac{\partial^2 J_1(X_1)}{\partial X_1^2} = -(p_1 + h_1^b) f_{D_1}(X_1) \quad (23)$$

$$\begin{aligned} \frac{\partial^2 J_2(X_2)}{\partial X_2^2} &= \frac{1}{\eta} (p_2 + r - c_e) f_{D_1} \left( \frac{X_2 - \delta^d}{\eta} \right) \\ &\quad - \frac{1}{\eta} (p_2 + r - v_f^b) f_{D_1} \left( \frac{X_2 - \delta^d}{\eta} \right) F_{D_2|\frac{X_2-\delta^d}{\eta}} \left( X_2 - \frac{X_2 - \delta^d}{\eta} \right) \\ &\quad - (p_2 + r - v_f^b) \int_0^{\frac{X_2 - \delta^d}{\eta}} f_{D_2|d_1}(X_2 - d_1) dF_{D_1}(d_1) \\ &= -(p_2 + r - v_f^b) \int_0^{\frac{X_2 - \delta^d}{\eta}} f_{D_2|d_1}(X_2 - d_1) dF_{D_1}(d_1) \end{aligned} \quad (24)$$

$$\begin{aligned} \frac{\partial^2 J_3(X_3)}{\partial X_3^2} &= -\frac{1}{\eta} (p_2 + r - c_e) f_{D_1} \left( \frac{X_3 - \delta^d}{\eta} \right) \\ &\quad + \frac{1}{\eta} (p_2 + r - v_f^b) F_{D_2|\frac{X_3-\delta^d}{\eta}} \left( X_3 - \frac{X_3 - \delta^d}{\eta} \right) f_{D_1} \left( \frac{X_3 - \delta^d}{\eta} \right) \\ &\quad - (p_2 + r - v_f^b) \int_{\frac{X_3 - \delta^d}{\eta}}^{X_3} f_{D_2|d_1}(X_3 - d_1) dF_{D_1}(d_1) \\ &= -(p_2 + r - v_f^b) \int_{\frac{X_3 - \delta^d}{\eta}}^{X_3} f_{D_2|d_1}(X_3 - d_1) dF_{D_1}(d_1) \end{aligned} \quad (25)$$

Clearly, all of the second partials are negative, and the result follows. ■

**Proof of Proposition 1:** In writing the expression for  $\Pi_{DS}^s(\cdot)$  we have implicitly assumed that the supplier will never produce more than what the buyer asks for using the expensive mode of production. This is certainly true whenever  $v_f^s \leq v_r^s + c_L(1 + \gamma)$ . Alternately, if  $v_f^s > v_r^s + c_L(1 + \gamma)$  then the supplier is motivated to turn all raw material to finished goods. However, if this is the case then the supplier will simply set  $X_L = X_3$  and will never produce anything using the more expensive mode of production. Both of these cases are captured in the profit function. Therefore, to find the optimal production level, we need to differentiate  $\Pi_{DS}^s$  with respect to  $X_L$  and equate this to zero.

$$\begin{aligned} \frac{\partial \Pi_{DS}^s}{\partial X_L} &= v_f^s - v_r^s - c_L - h^s - (v_f^s - v_r^s - c_L(1 - \gamma)) \left( 1 - F_{D_1} \left( \frac{X_L - \delta^d}{\eta} \right) \right) \\ &= c_L \gamma - h^s + (v_f^s - v_r^s - c_L(1 - \gamma)) F_{D_1} \left( \frac{X_L - \delta^d}{\eta} \right) \\ \frac{\partial^2 \Pi_{DS}^s}{\partial X_L^2} &= \frac{1}{\eta} (v_f^s - v_r^s - c_L(1 - \gamma)) f_{D_1} \left( \frac{X_L - \delta^d}{\eta} \right) \end{aligned}$$

Notice that the second partial will be negative for  $v_f^s - v_r^s - c_L(1 - \gamma) < 0$  and positive else. Hence,

$$F_{D_1} \left( \frac{\hat{X}_L^* - \delta^d}{\eta} \right) = \frac{c_L \gamma - h^s}{v_r^s + c_L(1 - \gamma) - v_f^s} \equiv \Phi(k_o)$$

so long as  $v_f^s < v_r^s + c_L(1 - \gamma)$ . Otherwise,  $k_o = \infty$  implying  $X_L^* = X_3$ . Hence,  $X_L^* = \max(X_2, \min(X_3, \hat{X}_L^*))$ .

**Proof of Lemma 1:** We will show this for the most general case, which corresponds to the fourth condition of the lemma. The proof for other cases are quite similar and are hence omitted. Consider when  $v_f^s > v_f^b$  and  $v_r^s + c_L(1 + \gamma) > v_f^s$ . In this case, the expected profit function for the last period corresponds to (8a) and (8b) which can be simplified as

$$\begin{aligned} \Pi_2^c(m_c, I_2^c) &= (p_2 - v_f^s)X_1^c + (v_f^s - v_r^s)X_L^c + v_r^s X_3^c + (p_2 + r - v_f^s)m_c - p_2(\mu_{D_2|d_1} + d_1) \\ &\quad + r(X_1^c - d_1)^+ + (v_f^s - v_r^s - c_L(1 + \gamma))(X_1^c + m_c - X_L^c)^+ \\ &\quad - (p_2 + r - v_f^b) \int_0^{X_1^c + m_c - d_1} (X_1^c + m_c - d_1 - d_2) dF_{D_2|d_1}(d_2). \end{aligned}$$

Let  $\mathbf{1}_x$  equal one if  $x$  is true and zero else. Taking partials with respect to  $m_c$ , we get

$$\begin{aligned} \frac{\partial \Pi_2^c(m_c, I_2^c)}{\partial m_c} &= p_2 + r - v_f^s + (v_f^s - v_r^s - c_L(1 + \gamma))\mathbf{1}_{m_c > X_L^c - X_1^c} - (p_2 + r - v_f^b)F_{D_2|d_1}(X_1^c + m_c - d_1) \\ \frac{\partial^2 \Pi_2^c(m_c, I_2^c)}{\partial m_c^2} &= -(p_2 + r - v_f^b)f_{D_2|d_1}(X_1^c + m_c - d_1) \end{aligned}$$

Hence, it is easy to see that  $\Pi_2^c(m_c, I_2^c)$  is concave and hence has a maximum where the first partial is equated to zero. That is,

$$F_{D_2|d_1}(X_1^c + m_c - d_1) = \frac{p_2 + r - v_f^s + (v_f^s - v_r^s - c_L(1 + \gamma))\mathbf{1}_{m_c > X_L^c - X_1^c}}{p_2 + r - v_f^b}.$$

This implies that the optimal value of  $m_c$  depends on how much was produced ahead of time, as well as the initial demand. Define  $k_{m_i}^c$  as follows.

$$\Phi(k_{m_1}^c) = \frac{p_2 + r - v_f^s}{p_2 + r - v_f^b} \quad \text{and} \quad \Phi(k_{m_2}^c) = \frac{p_2 + r - v_r^s - c_L(1 + \gamma)}{p_2 + r - v_f^b},$$

Hence,  $k_{m_1}^c$  corresponds to the optimal  $m_c$  should that value not exceed  $X_L^c - X_1^c$ . That is,  $m_c^* = d_1 + \mu_{D_2|d_1} - X_1^c + \sigma_{D_2|d_1} k_{m_1}^c = \eta d_1 + \delta_1^c - X_1^c$  should this not exceed  $X_L^c - X_1^c$ . This will happen whenever  $d_1 \leq \frac{X_L^c - \delta_1^c}{\eta}$ . Since the buyer cannot send units back, however, should  $F_{D_2|d_1}(X_1^c - d_1) < \frac{p_2 + r - v_f^s}{p_2 + r - v_f^b}$  (that is if  $d_1 \leq \frac{X_1^c - \delta_1^c}{\eta}$ ), then  $m_c^* = 0$ .

In a similar fashion, whenever  $d_1 \geq \frac{X_L^c - \delta_2^c}{\eta}$ , then  $m_c^* = d_1 + \mu_{D_2|d_1} - X_1^c + \sigma_{D_2|d_1} k_{m_2}^c = \eta d_1 + \delta_2^c - X_1^c$ . However, as the buyer cannot acquire more total units than  $X_3^c$  from the supplier, if  $F_{D_2|d_1}(X_3^c - d_1) > \frac{p_2 + r - v_r^s - c_L(1 + \gamma)}{p_2 + r - v_f^b}$  (that is if  $d_1 \geq \frac{X_3^c - \delta_2^c}{\eta}$ ), then  $m_c^* = X_3^c - X_1^c$ .

Therefore, only demands such that  $\frac{X_L^c - \delta_1^c}{\eta} < d_1 < \frac{X_L^c - \delta_2^c}{\eta}$  are unaccounted for. Such a demand outcome implies that  $F_{D_2|d_1}(X_L^c - d_1) > \frac{p_2 + r - v_f^s}{p_2 + r - v_f^b}$  but  $F_{D_2|d_1}(X_L^c - d_1) < \frac{p_2 + r - v_r^s - c_L(1 + \gamma)}{p_2 + r - v_f^b}$ . Hence, we set  $m_c^*$  equal to the boundary value,  $X_L^c - X_1^c$ .

Putting these scenarios together into one equation yields  $G(\delta_1^c, X_1^c) - G(\delta_1^c, X_L^c) + G(\delta_2^s, X_L^c) - G(\delta_2^s, X_3^s)$ .

**Proof of  $\Pi_{CS}(X_1^c, X_L^c, X_3^c)$ :** The proof is similar to proof of concavity of  $\Pi_{DS}^b(X_1, X_2, X_3)$  and hence omitted.  $\blacksquare$

**Proof of Proposition 2:**

**Part a.1:** When  $v_f^b$  exceeds both  $v_f^s$  and  $v_r^s + c_L + h^s$ , then the CS will convert all of the raw material purchased into finished goods using the cheaper production mode and then salvage all leftover finished goods at  $v_f^b$ . That is,  $X_L^{c*} = X_3^{c*}$  and  $m_c^* = X_3^{c*} - X_1^{c*} = X_L^{c*} - X_1^{c*}$ . For  $\rho < 1$ , this means that the profit function in the CS will be

$$\Pi_{CS}(X_1^c, X_L^c, X_3^c) = J_1^c(X_1^c) + J_L^c(X_L^c) \quad (26)$$

where  $J_1^c(X_1^c)$  is as in (11) and

$$J_L^c(X_L^c) = (p_2 + r - c_r - c_L - h^s)X_L^c - (p_2 + r - v_f^b) \int_0^{X_L^c} (X_L^c - d_{12}) dF_{D_{12}}(d_{12}), \quad (27)$$

where  $D_{12} = D_1 + D_2$  with a cdf of  $F_{D_{12}}(\cdot)$ .

From (9) and (11) we have that  $X_1^* = X_1^{c*}$  if  $c_2 - c_1 = h^s$ . If  $c_e \leq v_f^b$ , then  $m^* = X_3^* - X_2^*$  in the DS. This means that

$$\begin{aligned} \Pi_{DS}^b(X_1, X_2, X_3) &= J_1(X_1) + X_2(c_e + c_o - c_2) + X_3(p_2 + r - c_o - c_e) \\ &\quad - (p_2 + r - v_f^b) \int_0^{X_3} (X_3 - d_{12}) dF_{D_{12}}(d_{12}). \end{aligned} \quad (28)$$

Setting  $c_2 = c_o + c_e$  assures us that the buyer will buy as few firm orders as possible in the second period, and so  $X_2^* = X_1^*$  (an option good costs the same as a firm order good). If we set  $c_o + c_e = c_r + c_L + h^s$ , then  $X_L^{c*} = X_3^*$ , and so  $m_c^* = m^*$ . Clearly,  $X_3^* = X_3^{c*}$ . Substituting these values of prices and decisions, gives a supplier's profit function of

$$\Pi_{DS}^s(X_L, c_r + c_L, c_r + c_L + h^s, c_r + c_L + h^s - c_e, c_e) = X_L(c_L\gamma - h^s) - X_3(c_L\gamma - h^s). \quad (29)$$

Since  $h^s \leq c_L\gamma$  (Assumption C3), the supplier will set  $X_L = X_3$  and make zero profits. But  $X_3^* = X_L^{c*}$ , hence  $X_L^* = X_L^{c*}$ . Therefore, the decisions and profits of DS and CS are the same and we have channel coordination.

**Part a.2:** If  $v_f^s \leq v_f^b$  then in the CS all finished goods inventory will be salvaged at  $v_f^b$ . In addition, the first condition of Lemma 1 also holds and  $m_c^* = X_L^c - X_1^c + G(\delta_2^c, X_L^c) - G(\delta_2^c, X_3^c)$ . This means that the minimum number of goods that will be sent to the buyer in the second period is  $X_L^c - X_1^c$ . Furthermore, since  $v_r^s + c_L + h^s > v_f^b$ ,  $X_L^c < X_3^c$ . The profit function for the CS is given by  $\Pi_{CS}(X_1^c, X_L^c, X_3^c) = J_1^c(X_1^c) + J_L^c(X_L^c) + J_3^c(X_3^c)$  where  $J_1^c(X_1^c)$ ,  $J_L^c(X_L^c)$ , and  $J_3^c(X_3^c)$  are given by (11)–(13).

For channel coordination, we want the DS to mimic the CS by setting appropriate values of  $c_1$ ,  $c_2$ ,  $c_o$ , and  $c_e$ . From (11) we have that  $X_1^* = X_1^{c*}$  if  $c_2 - c_1 = h^s$ . With  $c_2 - c_o = c_L + v_r^s + h^s$  and  $c_e = v_r^s + c_L(1 + \gamma)$  we get that  $X_2^* = X_L^{c*}$ . In addition,  $c_e = v_r^s + c_L(1 + \gamma)$  implies that  $\delta^d = \delta_2^c$ , and so

$m_c^* = m^* + X_L^{c*} - X_1^{c*} = m^* + X_2^* - X_1^*$ . Substituting the value of  $c_e$  into (10) along with  $c_o = c_r - v_r^s$  and comparing with (13), we get that  $J_3(X_3) = J_3^c(X_3^c)$  giving  $X_3^* = X_3^{c*}$ .

Finally, we need to ensure that the supplier in the DS will also choose the CS's optimal production quantity as her production quantity; that is,  $X_L^* = X_L^{c*}$ . From Proposition 1, we know that, for the DS,  $X_L^* = \max(X_2, \min(X_3, \hat{X}_L^*))$ . For the transfer prices set as above, we have,

$$X_L^* = \max(X_2^*, \min(X_3^*, \hat{X}_L^*)) = \max(X_L^{c*}, \min(X_3^{c*}, \hat{X}_L^*)), \quad (30)$$

where  $\hat{X}_L^*$  satisfies

$$\frac{\partial \Pi_{DS}^s}{\partial X_L} = c_L \gamma - h^s + (v_f^s - v_r^s - c_L(1 + \gamma)) F_{D_1} \left( \frac{\hat{X}_L^* - \delta_2^c}{\eta} \right) = 0.$$

Now taking the derivative of (12) we get,

$$\begin{aligned} \frac{\partial J_L^c(X_L^c)}{\partial X_L^c} &= p_2 + r - v_r^s - c_L - h^s - (p_2 + r - v_r^s - c_L(1 + \gamma)) \left( 1 - F_{D_1} \left( \frac{X_L^c - \delta_2^c}{\eta} \right) \right) \\ &\quad - (p_2 + r - v_f^b) \int_0^{\frac{X_L^c - \delta_2^c}{\eta}} F_{D_2|d_1}(X_L^c - d_1) dF_{D_1}(d_1) \\ &\geq c_L \gamma - h^s + (p_2 + r - v_r^s - c_L(1 + \gamma)) F_{D_1} \left( \frac{X_L^c - \delta_2^c}{\eta} \right) \\ &\quad - (p_2 + r - v_f^b) \int_0^{\frac{X_L^c - \delta_2^c}{\eta}} dF_{D_1}(d_1) \\ &= c_L \gamma - h^s + (v_f^b - v_r^s - c_L(1 + \gamma)) F_{D_1} \left( \frac{X_L^c - \delta_2^c}{\eta} \right) \\ &\geq c_L \gamma - h^s + (v_f^s - v_r^s - c_L(1 + \gamma)) F_{D_1} \left( \frac{X_L^c - \delta_2^c}{\eta} \right) \\ &= \frac{\partial \Pi_{DS}^s}{\partial X_L} \Big|_{X_L = X_L^c}. \end{aligned}$$

Then,

$$\frac{\partial J_L^c(X_L^c)}{\partial X_L^c} \Big|_{X_L^c = X_L^{c*}} = 0 \geq \frac{\partial \Pi_{DS}^s}{\partial X_L} \Big|_{X_L = X_L^{c*}}.$$

Hence,  $X_L^* \geq \hat{X}_L^*$ . Since  $X_3^{c*} \geq X_L^{c*}$ , it follows, by substituting into (30), that  $X_L^* = X_L^{c*}$ . Thus we have shown all decisions in the CS and the DS are identical and we get channel coordination.

**Part a.3:** If  $v_f^s \geq v_r^s + c_L + h^s$ , the supplier will always produce all goods in the cheaper mode of production in both the CS and the DS. That is,  $X_L^* = X_3^*$  and  $X_L^{c*} = X_3^{c*}$ . Moreover,  $c_o + c_e = c_2$  and  $c_e > v_f^b$  imply that  $\frac{\partial J_2(X_2)}{\partial X_2} < 0$  for all  $X_2$  as shown below:

$$\begin{aligned} \frac{\partial J_2(X_2)}{\partial X_2} &= (p_2 + r - c_e) F_{D_1} \left( \frac{X_2 - \delta^d}{\eta} \right) \\ &\quad - (p_2 + r - v_f^b) \int_0^{\frac{X_2 - \delta^d}{\eta}} F_{D_2|d_1}(X_2 - d_1) dF_{D_1}(d_1) \\ &< (p_2 + r - c_e) F_{D_1} \left( \frac{X_2 - \delta^d}{\eta} \right) \end{aligned}$$

$$\begin{aligned}
& -(p_2 + r - v_f^b) \int_0^{\frac{X_2 - \delta^d}{\eta}} F_{D_2|d_1} \left( X_2 - \frac{X_2 - \delta^d}{\eta} \right) dF_{D_1}(d_1) \\
&= (p_2 + r - c_e) F_{D_1} \left( \frac{X_2 - \delta^d}{\eta} \right) - (p_2 + r - v_f^b) \int_0^{\frac{X_2 - \delta^d}{\eta}} \Phi(k_m) dF_{D_1}(d_1) \\
&= (p_2 + r - c_e) F_{D_1} \left( \frac{X_2 - \delta^d}{\eta} \right) - (p_2 + r - v_f^b) \Phi(k_m) F_{D_1} \left( \frac{X_2 - \delta^d}{\eta} \right) \\
&= 0
\end{aligned} \tag{31}$$

This implies that  $X_2^* = X_1^*$ . Substituting into the buyer's profit function and setting  $c_1 - c_o = v_f^s - h^s$  and  $c_e = v_f^s$  we get  $J_1(X_1) + J_2(X_1) = J_1^c(X_1)$ , where  $J_1^c(X_1)$  is as in (15). Therefore,  $X_1^* = X_1^{c*}$ . Since  $X_L^* = X_3^{c*}$ ,  $m_c^*$  is only a function of  $\delta_1^c$ . Finally,  $c_e = v_f^s$  implies that  $\delta_1^c = \delta^d$  and hence  $m^* = m_c^*$ .

Now, we only need to show that  $X_3^* = X_3^{c*}$ . Recall that  $X_L^* = X_3^{c*}$ . Substituting into the profit function for the CS, we get

$$\begin{aligned}
J_3^c(X_3^c) &= J_L^c(X_3^c) + J_3^c(X_3^c) \\
&= -(c_r + h^s - v_f^s + c_L) X_3 - (p_2 + r - v_f^s) \int_{\frac{X_3^c - \delta_1^c}{\eta}}^{\infty} (\delta_1^c + \eta d_1 - X_3^c) dF_{D_1}(d_1) \\
&\quad - (p_2 + r - v_f^b) \left( \int_{\frac{X_3^c - \delta_1^c}{\eta}}^{\infty} \int_0^{X_3^c - d_1} (X_3^c - d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \right. \\
&\quad \left. - \int_{\frac{X_3^c - \delta_1^c}{\eta}}^{\infty} \int_0^{\delta_1^c + (\eta-1)d_1} (\delta_1^c + (\eta-1)d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \right)
\end{aligned} \tag{32}$$

Since  $v_f^b \leq v_f^s$ ,  $\delta_1^c$  is finite. Setting  $c_o = c_r + c_L + h^s - v_f^s$  with  $c_e = v_f^s$  we get  $J_3(X_3) = J_3^c(X_3)$  and so  $X_3^* = X_3^{c*}$ . Hence, all of the order quantities will be the same, and we have channel coordination.

**Part b:** Consider the scenario where  $X_1^{c*} < X_L^{c*} < X_3^{c*}$ . Recall that  $v_f^s > v_f^b$  and  $v_r^s + c_L + h^s > v_f^s$ . This implies that  $\delta_1^c > \delta_2^c$  and the optimal number of options exercised,  $m_c^* = G(\delta_1^c, X_1^c) - G(\delta_1^c, X_L^c) + G(\delta_2^c, X_L^c) - G(\delta_2^c, X_3^c)$ . We can rewrite  $m_c^*$  as follows.

$$m_c^* = \begin{cases} 0, & \text{for } d_1 \leq \frac{X_1^c - \delta_1^c}{\eta} \\ \delta_1^c + \eta d_1 - X_1^c, & \text{for } \frac{X_1^c - \delta_1^c}{\eta} < d_1 \leq \frac{X_L^c - \delta_1^c}{\eta} \\ X_L^c - X_1^c, & \text{for } \frac{X_L^c - \delta_1^c}{\eta} < d_1 \leq \frac{X_L^c - \delta_2^c}{\eta} \\ \delta_2^c + \eta d_1 - X_1^c, & \text{for } \frac{X_L^c - \delta_2^c}{\eta} < d_1 \leq \frac{X_3^c - \delta_2^c}{\eta} \\ X_3^c - X_1^c, & \text{for } \frac{X_3^c - \delta_2^c}{\eta} < d_1 \end{cases}$$

Thus the number of options exercised depends on the first period demand being in one of the five distinct regions. The regions are distinct because  $\delta_1^c > \delta_2^c$  and  $X_1^{c*} < X_L^{c*} < X_3^{c*}$ . In the DS, however, the optimal number of options exercised depends on first period demand being in at most one of three regions; that is,

$$m^* = \begin{cases} 0, & \text{for } d_1 \leq \frac{X_2 - \delta^d}{\eta} \\ \delta^d + \eta d_1 - X_2, & \text{for } \frac{X_2 - \delta^d}{\eta} < d_1 \leq \frac{X_3 - \delta^d}{\eta} \\ X_3 - X_2, & \text{for } \frac{X_3 - \delta^d}{\eta} < d_1 \end{cases}$$

Since  $\delta^d$ , given by (3), is a function of  $c_e$  through  $k_m$ , the supplier may control the setting of region boundaries. There are, however, only two such boundaries available to influence in the DS. Hence,

there is no way to make  $m^* + X_2^* - X_1^* = m_c^*$  for every outcome  $d_1$ . Therefore, the DS cannot imitate the actions of the CS along every sample path, and so there will be no channel coordination. ■

**Proof of Proposition 3: Part a.1:** Shown in proof of Proposition 2, Part a.1.

**Part a.2:** The prices are  $c_1 = c_r + c_L$ ,  $c_2 = c_r + c_L + h^s$ ,  $c_o = c_r - v_r^s$ , and  $c_e = v_r^s + c_L(1 + \gamma)$  and the resulting decisions are  $X_1^* = X_1^{c*}$ ,  $X_2^* = X_L^* = X_L^{c*}$  and  $X_3^* = X_3^{c*}$ . The marginal cost of supplying up to  $X_1$  units of goods is  $c_r + c_L$  for which the supplier gets  $c_1$  per unit. The marginal cost of supplying up to  $X_2 - X_1$  units of goods is  $c_r + c_L + h^s$  for which the supplier gets  $c_2$  per unit. Furthermore,  $X_2^* = X_L^*$  implies that if any options are exercised in the DS, the supplier has to produce them in the expedited mode. Thus marginal cost of supplying goods against options is the higher labor cost  $c_L(1 + \gamma)$  plus the salvage value of the raw material  $v_r^s$ . Observe that at the beginning of second period when options are exercised, raw material has already been bought and hence the cost of raw material is essentially a sunk cost; all that matters is its salvage value. By setting  $c_e = v_r^s + c_L(1 + \gamma)$  the supplier supplies goods to the buyer against exercised options at marginal cost. Finally, the marginal cost of purchasing additional raw material is  $c_r - v_r^s$  which is also the price at which the supplier sells options to the buyer. Therefore, all transactions between the supplier and the buyer occur at marginal cost to the supplier and hence she makes zero profits.

**Part a.3:** The prices are  $c_1 = c_r + c_L$ ,  $c_2 = c_r + c_L + h^s$ ,  $c_o = c_r + c_L + h^s - v_f^s$ , and  $c_e = v_f^s$  and the resulting decisions are  $X_1^* = X_1^{c*} = X_2^*$ ,  $X_L^* = X_L^{c*} = X_3^* = X_3^{c*}$ . The marginal cost of supplying up to  $X_1$  units of goods is  $c_r + c_L$  for which the supplier gets  $c_1$  per unit. The marginal cost of supplying up to  $X_2 - X_1$  units of goods is  $c_r + c_L + h^s$  for which the supplier gets  $c_2$  per unit. Furthermore,  $X_3^* = X_L^*$  implies that if any options are exercised in the DS, the supplier has already produced them in the preliminary mode. By setting  $c_o + c_e = c_r + c_L + h^s$  the supplier supplies goods to the buyer against exercised options at marginal cost. Finally, the marginal cost of making an unused good is  $c_r + c_L + h^s - v_f^s$  which is also the price at which the supplier sells options to the buyer. Therefore, all transactions between the supplier and the buyer occur at marginal cost to the supplier and hence she makes zero profits. ■

#### Proof of Proposition 4

Let us consider a scenario where the supplier also offers to buy back any remaining finished goods the buyer may have, for a price  $c_b$ . However, it will cost  $t_{bs}$  per unit to have the goods returned. This is a charge the supplier incurs, though she may pass that on to the buyer through an appropriate  $c_b$ .

The buyer's expected profit function will remain the same as before, except that now,  $\max(v_f^b, c_b - h_2^b)$  will replace  $v_f^b$  everywhere it appears. That is, we get

$$\Pi_{DS}^b(X_1, X_2, X_3) = J_1(X_1) + J_2(X_2) + J_3(X_3),$$

where,

$$J_1(X_1) = -p_2\mu_2 - \mu_1(p_1 + p_2) - (p_1 + h_1^b) \int_0^{X_1} (X_1 - d_1) dF_{D_1}(d_1) + (p_1 + c_2 - c_1)X_1$$

$$\begin{aligned}
J_2(X_2) &= (p_2 + r - (c_2 - c_o))X_2 + (p_2 + r - c_e) \int_{\frac{X_2 - \delta^d}{\eta}}^{\infty} (\delta^d + \eta d_1 - X_2) dF_{D_1}(d_1) \\
&\quad - (p_2 + r - \max(v_f^b, c_b - h_2^b)) \int_0^{\frac{X_2 - \delta^d}{\eta}} \int_0^{X_2 - d_1} (X_2 - d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \\
&\quad - (p_2 + r - \max(v_f^b, c_b - h_2^b)) \int_{\frac{X_2 - \delta^d}{\eta}}^{\infty} \int_0^{\delta^d + (\eta-1)d_1} (\delta^d + (\eta-1)d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \\
J_3(X_3) &= -c_o X_3 - (p_2 + r - c_e) \int_{\frac{X_3 - \delta^d}{\eta}}^{\infty} (\delta^d + \eta d_1 - X_3) dF_{D_1}(d_1) \\
&\quad + (p_2 + r - \max(v_f^b, c_b - h_2^b)) \int_{\frac{X_3 - \delta^d}{\eta}}^{\infty} \int_0^{\delta^d + (\eta-1)d_1} (\delta^d + (\eta-1)d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \\
&\quad - (p_2 + r - \max(v_f^b, c_b - h_2^b)) \int_{\frac{X_3 - \delta^d}{\eta}}^{\infty} \int_0^{X_3 - d_1} (X_3 - d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1)
\end{aligned}$$

The supplier's profit function will now also need to incorporate the added salvage of finished goods which the buyer may return. That is, if  $v_f^b < c_b - h_2^b$ , then the supplier will also need to salvage  $X_2 + m^* - d_1 - d_2$ . For each of these units, she will get  $v_f^s - c_b - t_{bs}$ . Therefore, when this is the case, we can write the supplier's profit function as

$$\begin{aligned}
\Pi_{DS}^s(X_L, c_1, c_2, c_o, c_e) &= X_1(h^s - (c_2 - c_1)) + X_2(c_2 - c_o - v_f^s) - X_L(v_r^s - v_f^s + c_L + h^s) \\
&\quad + X_3(c_o + v_r^s - c_r) + (c_e - v_f^s) \int_{\frac{X_2 - \delta^d}{\eta}}^{\infty} (\delta^d - X_2 + \eta d_1) dF_{D_1}(d_1) \\
&\quad + (v_f^s - v_r^s - c_L(1 + \gamma)) \int_{\frac{X_L - \delta^d}{\eta}}^{\infty} (\delta^d - X_L + \eta d_1) dF_{D_1}(d_1) \\
&\quad + (v_r^s + c_L(1 + \gamma) - c_e) \int_{\frac{X_3 - \delta^d}{\eta}}^{\infty} (\delta^d - X_3 + \eta d_1) dF_{D_1}(d_1) \\
&\quad + (v_f^s - t_{bs} - c_b) \int_0^{\frac{X_2 - \delta^d}{\eta}} \int_0^{X_2 - d_1} (X_2 - d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \\
&\quad + (v_f^s - t_{bs} - c_b) \int_{\frac{X_2 - \delta^d}{\eta}}^{\infty} \int_0^{\delta^d + (\eta-1)d_1} (\delta^d + (\eta-1)d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \\
&\quad - (v_f^s - t_{bs} - c_b) \int_{\frac{X_3 - \delta^d}{\eta}}^{\infty} \int_0^{\delta^d + (\eta-1)d_1} (\delta^d + (\eta-1)d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \\
&\quad + (v_f^s - t_{bs} - c_b) \int_{\frac{X_3 - \delta^d}{\eta}}^{\infty} \int_0^{X_3 - d_1} (X_3 - d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1).
\end{aligned}$$

When  $v_f^b > c_b - h_2^b$ , then the last four terms will be omitted from the expression, and the supplier's profit will be the same as the non-buy back case. Note that the supplier's unconstrained production decision,  $\hat{X}_L^*$ , remains unaffected by whether or not the buyer will return goods.

In the centralized system, the buy back price is another transfer price and as such won't matter for the centralized system. However, under a buy back type arrangement, it now becomes possible for the finished goods to be salvaged wherever it is more lucrative. Hence, if costs are such that  $v_f^s - t_{bs} > \tilde{v}_f^b = v_f^b + h_2^b$  (i.e.,  $v_f^s - t_{bs} - h_2^b > v_f^b$ ), then the goods will be shipped back to the supplier

for salvage. Otherwise, they will be salvaged at the buyer. In any case,  $\bar{v} = \max(v_f^b, v_f^s - t_{bs} - h_2^b)$  will replace  $v_f^b$  in all CS profit equations.

Clearly, when  $v_f^s - t_{bs} - h_2^b < v_f^b$ , the CS will choose to salvage at the buyer. Hence, there is no change in the decisions made and general channel coordination is not possible. However, when  $v_f^s - t_{bs} - h_2^b \geq v_f^b$ , the CS is either indifferent between salvage locations (=) or prefers to salvage at the supplier (>), in which cases there might now be a new opportunity to channel coordinate. Therefore, let us consider this case.

**Part a.1:** Since  $v_f^s - h_2^b - t_{bs} \geq v_f^b$  and  $v_f^s \leq v_f^b$ , it must be true that  $h_2^b = t_{bs} = 0$  and  $v_f^s = v_f^b$ . When  $v_f^b$  equals  $v_f^s$  and exceeds  $v_r^s + c_L + h^s$ , then the CS will convert all of the raw material purchased into finished goods using the cheaper production mode and then salvage all leftover finished goods. It is indifferent between salvage locations, however. That is,  $X_L^{c*} = X_3^{c*}$  and  $m_c^* = X_3^{c*} - X_1^{c*} = X_L^{c*} - X_1^{c*}$ . This means that the profit function in the CS will be

$$\Pi_{CS}(X_1^c, X_L^c, X_3^c) = J_1^c(X_1^c) + J_L^c(X_L^c) \quad (33)$$

where  $J_1^c(X_1^c)$  is as in (11) and

$$J_L^c(X_L^c) = (p_2 + r - c_r - c_L - h^s)X_L^c - (p_2 + r - v_f^s) \int_0^{X_L^c} (X_L^c - d_{12}) dF_{D_{12}}(d_{12}), \quad (34)$$

where  $D_{12} = D_1 + D_2$  with a cdf of  $F_{D_{12}}(\cdot)$ .

From (9) and (11) we have that  $X_1^* = X_1^{c*}$  if  $c_2 - c_1 = h^s$ . If the supplier is to take advantage of the equal salvage access, she must convince the buyer to return goods. That is, she must set  $c_b > v_f^b$ . Then, if  $c_e \leq c_b$ , we have  $m^* = X_3^* - X_2^*$  in the DS. Hence, the supplier will turn all raw material into finished goods using the cheaper production mode, and so  $X_L^* = X_3^*$ . This also means that

$$\begin{aligned} \Pi_{DS}^b(X_1, X_2, X_3) &= J_1(X_1) + X_2(c_e + c_o - c_2) + X_3(p_2 + r - c_o - c_e) \\ &\quad - (p_2 + r - c_b) \int_0^{X_3} (X_3 - d_{12}) dF_{D_{12}}(d_{12}). \end{aligned} \quad (35)$$

Setting  $c_2 = c_o + c_e$  assures us that the buyer will buy as few firm orders as possible in the second period, and so  $X_2^* = X_1^*$  (an option good costs the same as a firm order good). Equating first order conditions for  $X_3$  and  $X_3^c$  give us

$$\frac{p_2 + r - c_2}{p_2 + r - c_b} = \frac{p_2 + r - c_r - c_L - h^s}{p_2 + r - v_f^s}.$$

Hence  $c_1 = p_2 + r - h^s - (p_2 + r - c_r - c_L - h^s) \frac{p_2 + r - c_b}{p_2 + r - v_f^s}$  will assure that  $X_3^* = X_3^{c*}$ , and we will coordinate the channel.

**Part a.2:** Since  $v_f^s - h_2^b - t_{bs} \geq v_f^b$  and  $v_f^s \leq v_f^b$ , it must be true that  $h_2^b = t_{bs} = 0$  and  $v_f^s = v_f^b$ , and so all finished goods inventory will be salvaged at  $v_f^b$ . In addition, the first condition of Lemma 1 also holds and  $m_c^* = X_L^c - X_1^c + G(\delta_2^c, X_L^c) - G(\delta_2^c, X_3^c)$ . This means that the minimum number of goods that will be sent to the buyer in the second period is  $X_L^c - X_1^c$ . Furthermore, since  $v_r^s + c_L + h^s > v_f^b$ ,  $X_L^c < X_3^c$ .

The profit function for the CS is given by  $\Pi_{CS}(X_1^c, X_L^c, X_3^c) = J_1^c(X_1^c) + J_L^c(X_L^c) + J_3^c(X_3^c)$  where  $J_1^c(X_1^c)$ ,  $J_L^c(X_L^c)$ , and  $J_3^c(X_3^c)$  are given by (11)–(13).

For channel coordination, we want the DS to mimic the CS by setting appropriate values of  $c_1$ ,  $c_2$ ,  $c_b$ ,  $c_o$ , and  $c_e$ . To get the buyer to ship goods back, the supplier needs to make sure that  $c_b > v_f^b$ . From (11) we have that  $X_1^* = X_1^{c*}$  if  $c_2 - c_1 = h^s$ . To get  $m^* + X_2 - X_1 = m_c^*$  we also need  $\delta^d = \delta_2^c$  and  $X_2^* = X_L^{c*}$ . To get identical deltas, we must have

$$\frac{p_2 + r - c_e}{p_2 + r - c_b} = \frac{p_2 + r - v_r^s - c_L(1 + \gamma)}{p_2 + r - v_f^s}.$$

First order conditions for  $X_2$  and  $X_L^c$  also indicate that we must have

$$\frac{p_2 + r - c_2 + c_o}{p_2 + r - c_b} = \frac{p_2 + r - v_r^s - c_L - h^s}{p_2 + r - v_f^s}$$

in order to have  $X_2 = X_L^c$ .

To get  $X_3^* = X_3^{c*}$ , first order conditions tell us that we need

$$\frac{c_o}{p_2 + r - c_b} = \frac{c_r - v_r^s}{p_2 + r - v_f^s}.$$

Finally, we need to ensure that the supplier in the DS will also choose the CS's optimal production quantity as her production quantity; that is,  $X_L^* = X_L^{c*}$ . This is the exact same situation as Proposition 2, Part a.2, and from that proof we know that  $X_L^{c*} \geq \hat{X}_L^*$ . Since  $X_3^* \geq X_L^*$ , it follows that  $X_L^* = X_L^{c*}$ . Thus we have shown all decisions in the CS and the DS are identical and we get channel coordination.

**Part a.3:** Here,  $v_f^s - h_2^b - t_{bs} \geq v_f^b$  and  $v_f^s > v_f^b$  imply that  $v_f^s - h_2^b - t_{bs} > v_f^b$ . Now, if  $v_f^s \geq v_r^s + c_L + h^s$ , the supplier will always produce all goods in the cheaper mode of production in both the CS and the DS. That is,  $X_L^* = X_3^*$  and  $X_L^{c*} = X_3^{c*}$ . Then, the profit function for the CS appears as in (15)–(16). For channel coordination, we want the DS to mimic the CS by setting appropriate values of  $c_1$ ,  $c_2$ ,  $c_b$ ,  $c_o$ , and  $c_e$ . To get the buyer to ship goods back, the supplier needs to make sure that  $c_b - h_2^b > v_f^b$ .

To get  $m^* + X_2 - X_1 = m_c^*$ , we need to have  $\delta^d = \delta_1^c$ , and this will happen when

$$\frac{p_2 + r - c_e}{p_2 + r - c_b + h_2^b} = \frac{p_2 + r - v_r^s - c_L(1 + \gamma)}{p_2 + r - v_f^s + h_2^b + t_{bs}}.$$

To get  $X_3 = X_3^c$ , we need to consider the first order condition of (16). This indicates that we need

$$\frac{c_o}{p_2 + r - c_b + h_2^b} = \frac{c_r + c_L + h^s - v_f^s}{p_2 + r - v_f^s + h_2^b + t_{bs}}.$$

To get  $X_1 = X_1^c$ , we first need to have  $X_1 = X_2$ . This will certainly happen when  $c_o + c_e = c_2$ . Define  $Z(X)$  as follows.

$$Z(X) = \Phi(k_{m_1}^c) \left( 1 - F_{D_1} \left( \frac{X - \delta_1^c}{\eta} \right) \right) + \int_0^{\frac{X - \delta_1^c}{\eta}} F_{D_2|d_1}(X - d_1) dF_{D_1}(d_1).$$

Then, the first order conditions of (15) and the profit function for the DS, give us

$$p_1 + p_2 + r - v_f^s + h^s - (h_1^b + p_1)F_{D_1}(X_1^c) - (p_1 + r - v_f^s + t_{bs} + h_2^b)\mathcal{Z}(X_1^c) = 0$$

and

$$p_1 + p_2 + r - c_1 + c_o - (h_1^b + p_1)F_{D_1}(X_1) - (p_1 + r - c_b + h_2^b)\mathcal{Z}(X_1) = 0$$

Hence, to get  $X_1^* = X_1^{c*}$ , we need  $c_1 = c_o + v_f^s - h^s + \mathcal{Z}(X_1^{c*})(c_b - v_f^s + h^s)$ .

Therefore, all of the order quantities will be the same, and we have channel coordination. ■

**Part b:** Same as proof of Proposition 2, Part b.

### Proof of Proposition 5

We wish to show that the supplier can operate in an individually rational manner and still coordinate the supply chain by using a return policy.

**Part a.1:** In this case,  $m^* = X_2^* - X_3^*$ ,  $X_1^* = X_2^*$  and  $X_L^* = X_3^*$ , we can write the supplier's profit function as follows.

$$\begin{aligned} \Pi_{DS}^s(X_L^*, c_1, c_2, c_o, c_e) &= c_1 X_1^* + c_2 (X_2^* - X_1^*) + c_o (X_3^* - X_2^*) + c_e (X_3^* - X_2^*) - h^s (X_L^* - X_1^*) \\ &\quad - c_r X_3^* - c_L X_L^* + (v_f^s - c_b) \int_0^{X_3^*} (X_3^* - d_{12}) dF_{D_1+D_2}(d_{12}) \\ &= c_1 X_1^* + (c_o + c_e)(X_3^* - X_1^*) - h^s (X_3^* - X_1^*) - (c_r + c_L) X_3^* \\ &\quad + (v_f^s - c_b) \int_0^{X_3^*} (X_3^* - d_{12}) dF_{D_1+D_2}(d_{12}) \end{aligned} \quad (36)$$

Then, noting that  $c_e + c_o = c_2 = c_1 + h^s$  and  $c_1 = (p_2 + r - h^s)(1 - Y) + (c_r + c_L)Y$  in the pricing scheme and that  $v_f^s = v_f^b$ , we can further simply and get

$$\begin{aligned} \Pi_{DS}^s(X_L^*, c_1, c_2, c_o, c_e) &= (c_1 - c_r - c_L) X_3^* + (v_f^s - c_b) \int_0^{X_3^*} (X_3^* - d_{12}) dF_{D_1+D_2}(d_{12}) \\ &= (1 - Y) \left[ (p_2 + r - h^s - c_r - c_L) X_3^* - (p_2 + r - v_f^b) \int_0^{X_3^*} (X_3^* - d_{12}) dF_{D_1+D_2}(d_{12}) \right] \\ &= (1 - Y) J_L^c(X_3^*) \end{aligned} \quad (37)$$

However,  $X_3^* = X_L^* = X_L^{c*}$  with our current prices. Moreover, since  $c_b > v_f^b + h_2^b$ , we know that  $Y \leq 1$ . Therefore, the supplier will make non-negative profits if  $J_L^c(X_L^{c*}) \geq 0$ . First note that the first order conditions of  $J_L^c(X_L^c)$  indicate that  $F_{D_1+D_2}(X_L^{c*}) = \frac{p_2+r-c_r-c_L-h^s}{p_2+r-v_f^s}$ . Substituting this into  $J_L^c(X_L^c)$  gives us

$$\begin{aligned} J_L^c(X_L^c) &= (p_2 + r - c_r - c_L - h^s) X_L^c - (p_2 + r - v_f^s) \int_0^{X_L^c} (X_L^c - d_{12}) dF_{D_1+D_2}(d_{12}) \\ &= (p_2 + r - c_r - c_L - h^s) X_L^c - (p_2 + r - v_f^s) X_L^c F_{D_1+D_2}(X_L^c) + (p_2 + r - v_f^s) \int_0^{X_L^c} d_{12} dF_{D_1+D_2}(d_{12}) \\ &= (p_2 + r - v_f^s) \int_0^{X_L^c} d_{12} dF_{D_1+D_2}(d_{12}) \\ &\geq 0 \end{aligned}$$

Therefore, the supplier makes  $(1-Y)J_L^c(X_L^{c*})$ . Moreover, the larger she sets  $c_b$ , the more she makes, up to the system profits less the buyer's reservation profit.

**Part a.2:** In this case, we have that  $X_2 = X_L = X_L^*$ . This means that the supplier's profit function can be written as

$$\begin{aligned}
\Pi_{DS}^s(X_L, c_1, c_2, c_o, c_e) &= X_1(h^s - (c_2 - c_1)) + X_L(c_2 - c_o - v_r^s - c_L - h^s) + X_3(c_o + v_r^s - c_r) \\
&\quad + (c_e - v_r^s - c_L(1 + \gamma)) \int_{\frac{X_L - \delta^d}{\eta}}^{\infty} (\delta^d - X_L + \eta d_1) dF_{D_1}(d_1) \\
&\quad + (v_r^s + c_L(1 + \gamma) - c_e) \int_{\frac{X_3 - \delta^d}{\eta}}^{\infty} (\delta^d - X_3 + \eta d_1) dF_{D_1}(d_1) \\
&\quad + (v_f^s - c_b) \int_0^{\frac{X_L - \delta^d}{\eta}} \int_0^{X_L - d_1} (X_L - d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \\
&\quad + (v_f^s - c_b) \int_{\frac{X_L - \delta^d}{\eta}}^{\infty} \int_0^{\delta^d + (\eta-1)d_1} (\delta^d + (\eta-1)d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \\
&\quad - (v_f^s - c_b) \int_{\frac{X_3 - \delta^d}{\eta}}^{\infty} \int_0^{\delta^d + (\eta-1)d_1} (\delta^d + (\eta-1)d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \\
&\quad + (v_f^s - c_b) \int_{\frac{X_3 - \delta^d}{\eta}}^{\infty} \int_0^{X_3 - d_1} (X_3 - d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1).
\end{aligned}$$

Substituting in for the wholesale, option, and exercise prices, we get that  $c_2 - c_o - v_r^s - c_L - h^s = (1-Y)(p_2 + r - h^s - c_L - v_r^s)$ ,  $c_o + v_r^s - c_r = (1-Y)(v_r^s - c_r)$ , and  $c_e - v_r^s - c_L(1 + \gamma) = (1-Y)(p_2 + r - v_r^s - c_L(1 + \gamma))$ . This means that we can rewrite the supplier's profit function as

$$\begin{aligned}
\Pi_{DS}^s(X_L, c_1, c_2, c_o, c_e) &= X_L(1 - Y)(p_2 + r - h^s - c_L - v_r^s) + X_3(1 - Y)(v_r^s - c_r) \\
&\quad + (1 - Y)(p_2 + r - v_r^s - c_L(1 + \gamma)) \int_{\frac{X_L - \delta^d}{\eta}}^{\infty} (\delta^d - X_L + \eta d_1) dF_{D_1}(d_1) \\
&\quad - (1 - Y)(p_2 + r - v_r^s - c_L(1 + \gamma)) \int_{\frac{X_3 - \delta^d}{\eta}}^{\infty} (\delta^d - X_3 + \eta d_1) dF_{D_1}(d_1) \\
&\quad + (1 - Y)(p_2 + r - v_f^b) \int_0^{\frac{X_L - \delta^d}{\eta}} \int_0^{X_L - d_1} (X_L - d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \\
&\quad + (1 - Y)(p_2 + r - v_f^b) \int_{\frac{X_L - \delta^d}{\eta}}^{\infty} \int_0^{\delta^d + (\eta-1)d_1} (\delta^d + (\eta-1)d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \\
&\quad - (1 - Y)(p_2 + r - v_f^b) \int_{\frac{X_3 - \delta^d}{\eta}}^{\infty} \int_0^{\delta^d + (\eta-1)d_1} (\delta^d + (\eta-1)d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \\
&\quad + (1 - Y)(p_2 + r - v_f^b) \int_{\frac{X_3 - \delta^d}{\eta}}^{\infty} \int_0^{X_3 - d_1} (X_3 - d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \\
&= (1 - Y) [J_L^c(X_L) + J_3^c(X_3)]
\end{aligned}$$

where  $J_L^c(\cdot)$  and  $J_3^c(\cdot)$  are as in (12) and (13). Since  $X_L^* = X_L^{c*}$  and  $X_3^* = X_3^{c*}$ , we only need to show that  $J_L^c(X_L^{c*}) + J_3^c(X_3^{c*}) \geq 0$  for the supplier to have non-negative profits. Using the first order condition

of (12) we can rewrite  $J_L^c(X_L^{c*})$  as follows.

$$\begin{aligned}
J_L^c(X_L^{c*}) &= (p_2 + r - v_r^s - c_L - h^s)X_L^{c*} - (p_2 + r - v_r^s - c_L(1 + \gamma)) \left(1 - F_{D_1} \left(\frac{X_L^{c*} - \delta_2^c}{\eta}\right)\right) X_L^{c*} \\
&\quad + (p_2 + r - v_r^s - c_L(1 + \gamma)) \int_{\frac{X_L^{c*} - \delta_2^c}{\eta}}^{\infty} (\delta_2^c + \eta d_1) dF_{D_1}(d_1) \\
&\quad - (p_2 + r - v_f^b) \left[ \int_0^{\frac{X_L^{c*} - \delta_2^c}{\eta}} X^{c*} F_{D_2|d_1}(X_L^c - d_1) dF_{D_1}(d_1) \right. \\
&\quad - \int_0^{\frac{X_L^{c*} - \delta_2^c}{\eta}} \int_0^{X_L^c - d_1} (d_1 + d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \\
&\quad \left. + \int_{\frac{X_L^c - \delta_2^c}{\eta}}^{\infty} \int_0^{\delta_2^c + (\eta-1)d_1} (\delta_2^c + (\eta-1)d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \right] \\
&= \frac{\partial J_L^c(X_L^c)}{\partial X_L^c} \Big|_{X_L^{c*}} + (p_2 + r - v_r^s - c_L(1 + \gamma)) \int_{\frac{X_L^{c*} - \delta_2^c}{\eta}}^{\infty} (\delta_2^c + \eta d_1) dF_{D_1}(d_1) \\
&\quad + (p_2 + r - v_f^b) \left[ \int_0^{\frac{X_L^{c*} - \delta_2^c}{\eta}} \int_0^{X_L^c - d_1} (d_1 + d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \right. \\
&\quad \left. - \int_{\frac{X_L^c - \delta_2^c}{\eta}}^{\infty} \int_0^{\delta_2^c + (\eta-1)d_1} (\delta_2^c + (\eta-1)d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \right]
\end{aligned}$$

Similarly,

$$\begin{aligned}
J_3^c(X_3^{c*}) &= \frac{\partial J_3^c(X_3^c)}{\partial X_3^c} \Big|_{X_3^{c*}} - (p_2 + r - v_r^s - c_L(1 + \gamma)) \int_{\frac{X_3^{c*} - \delta_2^c}{\eta}}^{\infty} (\delta_2^c + \eta d_1) dF_{D_1}(d_1) \\
&\quad + (p_2 + r - v_f^b) \left[ \int_{\frac{X_3^{c*} - \delta_2^c}{\eta}}^{\infty} \int_0^{X_3^c - d_1} (d_1 + d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \right. \\
&\quad \left. + \int_{\frac{X_3^c - \delta_2^c}{\eta}}^{\infty} \int_0^{\delta_2^c + (\eta-1)d_1} (\delta_2^c + (\eta-1)d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \right]
\end{aligned}$$

Therefore, we get

$$\begin{aligned}
J_L^c(X_L^{c*}) + J_3^c(X_3^{c*}) &= (p_2 + r - v_r^s - c_L(1 + \gamma)) \int_{\frac{X_L^{c*} - \delta_2^c}{\eta}}^{\frac{X_3^{c*} - \delta_2^c}{\eta}} (\delta_2^c + \eta d_1) dF_{D_1}(d_1) \\
&\quad + (p_2 + r - v_f^b) \left[ \int_{\frac{X_3^{c*} - \delta_2^c}{\eta}}^{\infty} \int_0^{X_3^c - d_1} (d_1 + d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \right. \\
&\quad + \int_0^{\frac{X_L^{c*} - \delta_2^c}{\eta}} \int_0^{X_L^c - d_1} (d_1 + d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \\
&\quad - \int_{\frac{X_L^c - \delta_2^c}{\eta}}^{\frac{X_3^c - \delta_2^c}{\eta}} (\delta_2^c + \eta d_1) F_{D_2|d_1}(\delta_2^c + (\eta-1)d_1) dF_{D_1}(d_1) \\
&\quad \left. + \int_{\frac{X_L^c - \delta_2^c}{\eta}}^{\frac{X_3^c - \delta_2^c}{\eta}} \int_0^{\delta_2^c + (\eta-1)d_1} (d_1 + d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \right]
\end{aligned}$$

However, since  $p_2 + r - v_f^s - c_L(1 + \gamma) \geq p_2 + r - v_f^b$  and  $F_{D_2|d_1}(\delta_2^c + (\eta - 1)d_1) \leq 1$ , we have that the sum is greater than or equal to zero, and hence the supplier makes non-negative profits.

**Part a.3:** In this case,  $X_L^* = X_3^* = X_L^{c^*} = X_3^{c^*}$ , and  $X_1^* = X_2^*$ , and so we get

$$\begin{aligned} \Pi_{DS}^s(X_L, c_1, c_2, c_o, c_e) &= X_1(h^s + c_1 - c_o - v_f^s) - X_3(c_r + c_L + h^s - c_o - v_f^s) \\ &\quad + (c_e - v_f^s) \int_{\frac{X_1 - \delta^d}{\eta}}^{\infty} (\delta^d - X_1 + \eta d_1) dF_{D_1}(d_1) \\ &\quad + (v_f^s - c_e) \int_{\frac{X_3 - \delta^d}{\eta}}^{\infty} (\delta^d - X_3 + \eta d_1) dF_{D_1}(d_1) \\ &\quad + (v_f^s - t_{bs} - c_b) \int_0^{\frac{X_L - \delta^d}{\eta}} \int_0^{X_2 - d_1} (X_2 - d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \\ &\quad + (v_f^s - t_{bs} - c_b) \int_{\frac{X_2 - \delta^d}{\eta}}^{\infty} \int_0^{\delta^d + (\eta - 1)d_1} (\delta^d + (\eta - 1)d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \\ &\quad - (v_f^s - t_{bs} - c_b) \int_{\frac{X_3 - \delta^d}{\eta}}^{\infty} \int_0^{\delta^d + (\eta - 1)d_1} (\delta^d + (\eta - 1)d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \\ &\quad + (v_f^s - t_{bs} - c_b) \int_{\frac{X_3 - \delta^d}{\eta}}^{\infty} \int_0^{X_3 - d_1} (X_3 - d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1). \end{aligned}$$

Using the optimal prices, we get that  $h^s + c_1 - c_o - v_f^s = (c_b - v_f^s + t_{bs})\mathcal{Z}(X_1^{c^*}) = (1 - Y)(p_2 + r + h_2^s + t_{bs} - v_f^s)\mathcal{Z}(X_1^{c^*})$ ,  $c_r + c_L + h^s - c_o - v_f^s = (1 - Y)(c_r + c_L + h^s - v_f^s)$  and  $c_e - v_f^s = (1 - Y)(p_2 + r - v_f^s)$ .

Substituting in gives us

$$\begin{aligned} \Pi_{DS}^s &= X_1(1 - Y)(p_2 + r + h_2^s + t_{bs} - v_f^s)\mathcal{Z}(X_1^{c^*}) - X_3(1 - Y)(c_r + c_L + h^s - v_f^s) \\ &\quad + (1 - Y)(p_2 + r - v_f^s) \int_{\frac{X_1 - \delta^d}{\eta}}^{\infty} (\delta^d - X_1 + \eta d_1) dF_{D_1}(d_1) \\ &\quad - (1 - Y)(p_2 + r - v_f^s) \int_{\frac{X_3 - \delta^d}{\eta}}^{\infty} (\delta^d - X_3 + \eta d_1) dF_{D_1}(d_1) \\ &\quad + (1 - Y)(p_2 + r + h_2^b + t_{bs} - v_f^s) \int_0^{\frac{X_1 - \delta^d}{\eta}} \int_0^{X_1 - d_1} (X_1 - d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \\ &\quad + (1 - Y)(p_2 + r + h_2^b + t_{bs} - v_f^b) \int_{\frac{X_1 - \delta^d}{\eta}}^{\infty} \int_0^{\delta^d + (\eta - 1)d_1} (\delta^d + (\eta - 1)d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \\ &\quad - (1 - Y)(p_2 + r + h_2^b + t_{bs} - v_f^b) \int_{\frac{X_3 - \delta^d}{\eta}}^{\infty} \int_0^{\delta^d + (\eta - 1)d_1} (\delta^d + (\eta - 1)d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \\ &\quad + (1 - Y)(p_2 + r + h_2^b + t_{bs} - v_f^b) \int_{\frac{X_3 - \delta^d}{\eta}}^{\infty} \int_0^{X_3 - d_1} (X_3 - d_1 - d_2) dF_{D_2|d_1}(d_2) dF_{D_1}(d_1) \\ &= (1 - Y) \left[ J_1^c(X_1^{c^*}) + \mu_2 p_2 + \mu_1(p_1 + p_2) - (h_1^b + p_1) \int_0^{X_1^{c^*}} d_1 dF_{D_1}(d_1) \right] + (1 - Y) J_3^c(X_3) \\ &= (1 - Y) \left[ J_1^c(X_1^{c^*}) + J_3^c(X_3) + \mu_2 p_2 + \mu_1(p_1 + p_2) - (h_1^b + p_1)(\mu_1 F_{D_1}(X_1^{c^*}) - \sigma_1^2 f_{D_1}(X_1^{c^*})) \right] \\ &= (1 - Y) \left[ J_1^c(X_1^{c^*}) + J_3^c(X_3) + (\mu_1 + \mu_2)p_2 + \mu_1(p_1 - (h_1^b + p_1)F_{D_1}(X_1^{c^*})) + (h_1^b + p_1)\sigma_1^2 f_{D_1}(X_1^{c^*}) \right] \end{aligned}$$

Since  $F_{D_1}(X_1^{c^*}) \leq \frac{p_1}{p_1 + h_1^b}$ , we have that the supplier will make non-negative profits. ■